Mechanical Systems and Signal Processing 54-55 (2015) 481–490

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Time-varying torsional stiffness identification on a vertical beam using Chebyshev polynomials

François Martel [a,*], Denis Rancourt [a], Catherine Chochol [b], Yves St-Amant [c], Simon Chesne [b], Didier Rémond [b]

[a] Département de Génie Mécanique, Université de Sherbrooke, Sherbrooke, Canada J1K 2R1
[b] Université de Lyon, CNRS UMR5259, LaMCoS, INSA-Lyon, F-69621 Villeurbanne, France
[c] Département de Génie Mécanique, Université Laval, Québec, Canada G1V 0A6

ARTICLE INFO

Article history:
Received 26 May 2014
Received in revised form
10 August 2014
Accepted 6 September 2014
Available online 26 September 2014

Keywords:
Linear time-varying systems
Parameter identification
Chebyshev polynomials
Dynamic systems
Experimental mechanical systems
Modal analysis

ABSTRACT

This paper investigates the performance of the Chebyshev polynomial basis to identify the time-varying mechanical impedance of a vertical beam in torsion. The projection, derivation and product properties of Chebyshev polynomials were used to linearize the differential equation of 1-DOF mechanical systems having multiple time-varying parameters. This allowed the identification of a reduced set of projection coefficients without prior knowledge of initial system states conditions. The method was then applied to experimental data obtained from an equilateral beam excited in torsion while one beam support location was changed over time. Results showed $6.62\times10^{-2}\%$ error in stiffness predictions compared to theoretical estimates. Signal filtering was critical to avoid contamination by bending modes of the beam and prior knowledge of inertia led to better results.

1. Introduction

Numerous applications require parameter estimation of continuous time systems. This important and broadly reviewed field [1,2] is often used on discrete time data issued from experimental systems or numerical simulations. Linear time-varying (LTV) systems are getting more attention due to more sophisticated systems and models. Identification of the time-varying parameters of LTV systems can be achieved via various approaches, such as ensemble average of impulses responses [3], parallel-cascade algorithm [4], or wavelet-based methods [5].

An alternative approach is to identify time-varying parameters by estimating the coefficients of their projection on an orthogonal basis [6,7]. Some commonly used basis include Legendre series [8,9], block-pulse functions [10], Fourier series [11] and Laguerre polynomials [12]. The Chebyshev series approximation were also used for time-varying [13–15] parameter estimation and were reported to have certain advantages over other orthogonal series since they are an almost uniform approximation in the time interval of interest [16]. However, methods employing Chebyshev polynomials were, to our knowledge, formulated via integrals and thus require states initial conditions to be known or identified simultaneously. This was until a parameter estimation method based on Chebyshev basis and formulated via signal derivatives instead of integrals was proposed and applied to nonlinear systems in [17]. The method was also applied on continuous structures in

* Corresponding author. Tel.: +1 819 821 8000x63160; fax: +1 819 821 7163.
E-mail addresses: francois.martel2@usherbrooke.ca (F. Martel), denis.rancourt@usherbrooke.ca (D. Rancourt), catherine.chochol@insa-lyon.fr (C. Chochol), yves.st-amant@gmc.ulaval.ca (Y. St-Amant), simon.chesne@insa-lyon.fr (S. Chesne), didier.remond@insa-lyon.fr (D. Rémond).

http://dx.doi.org/10.1016/j.ymssp.2014.09.004
0888-3270/

[18] with an improved derivation operator. A similar approach was also proposed by [19] to identify constant parameters within nonlinear time-varying dynamical systems.

This paper adapts the method proposed in [17] to identify time-varying parameters and validates its performance using experimental results on a torsional beam. Some properties of the Chebyshev basis that are required to develop the identification method are first presented. Amongst other properties and as any polynomial basis, Chebyshev basis include a matrix representation of its coefficients and a linear relationship for both derivation and multiplication operations [20]. Thanks to these properties, the differential equation of a 1DOF dynamical system having time-varying parameters is linearized so that its unknown coefficients are estimated by means of a standard least-square algorithm. Experimental results are then provided to validate the method by investigating its ability identify a continuous mechanical system with time-varying stiffness, constant inertia and negligible damping. Calibration of the apparatus is presented before parameter identification is performed. The performance of the method is then analyzed with regards the need for fixing two identification parameters beforehand. The signal filtering process is investigated in details in order to deal with the multi vibration modes of the continuous system and, in particular, the potential cross-coupling between torsional and bending modes as well as system excitation by a moving load on the bar.

2. Methods

2.1. Chebyshev polynomials properties

2.1.1. Projection

The n order Chebyshev polynomial is defined in the interval $\tau = [-1, +1]$ by the following equation:

$$T_n(\tau) = \cos(n\, arccos(\tau)). \tag{1}$$

Any function $x(\tau)$ can be expanded on the n order Chebyshev basis $\{T^n(\tau)\} = \langle T_0(\tau) \quad T_1(\tau) \quad T_2(\tau) \quad \dots \quad T_n(\tau)\rangle^T$ as:

$$\begin{aligned} x(\tau) &\simeq \langle x_0 \quad x_1 \quad x_i \quad \dots \quad x_n\rangle \cdot \{T^n(\tau)\}, \\ x(\tau) &\simeq \langle x_C\rangle \cdot \{T^n(\tau)\}, \end{aligned} \tag{2}$$

where $\langle x_C\rangle$ contains the x_i coordinates of $x(\tau)$ in the basis $\{T^n(\tau)\}$.

If τ is sampled on N_e discrete points of index m so that the function $x(\tau)$ is discretized in the vector $\langle x(m)\rangle_{N_e}$, (2) becomes:

$$\langle x(m)\rangle_{N_e} \simeq \langle x_C\rangle_{n+1} \cdot [T^n(m)]_{n+1,\, N_e} \tag{3}$$

and the projection coefficients can be estimated using a least-square approximation:

$$\langle x_C\rangle \simeq \langle x(m)\rangle \cdot [T^n(m)]^T \cdot \left([T^n(m)] \cdot [T^n(m)]^T\right)^{-1}. \tag{4}$$

2.1.2. Derivation

Since the derivative of each Chebyshev polynomial can be evaluated by a linear combination of lower order polynomials, the time derivative of a function can be expressed as:

$$\frac{dx}{dt} = \langle x_C\rangle_{n+1} \cdot [D]_{n+1,\, n+1} \cdot [T^n(m)]_{n+1,\, N_e}, \tag{5}$$

where [D] is the derivative operator described in [17]. This operator gives increasing importance to higher order polynomials because the linear combination coefficients in [D] are proportional to 2n. The sensibility of the derivation error to a projection error is thus higher with larger order polynomials. This point can be largely improved by the use of Chebyshev polynomial properties as demonstrated in [18] but the first level approach proposed in this paper is precise enough to solve the LTV problem.

2.1.3. Product

If two polynomials $P(\tau)$ and $Q(\tau)$ are expressed in the n order Chebyshev basis as:

$$\begin{aligned} P(\tau) &= \sum_{k=0}^{n} p_k T_k(\tau), \\ Q(\tau) &= \sum_{k=0}^{n} q_k T_k(\tau), \end{aligned} \tag{6}$$

then their product can be expressed in the $2n$ order basis:

$$P(\tau) \cdot Q(\tau) = \sum_{i=0}^{2n} a_i T_i(\tau), \tag{7}$$