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Input load identification from optimally placed strain gages using D-optimal design and model reduction

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ABSTRACT

This paper presents a time domain technique for estimating dynamic loads acting on a structure from strain time response measured at a finite number of optimally placed strain gages on the structure. The technique utilizes model reduction to obtain precise load estimates. The structure essentially acts as its own load transducer. The approach is based on the fact that the strain response of an elastic vibrating system can be expressed as a linear superposition of its strain modes. Since the strain modes as well as the normal displacement modes are intrinsic dynamic characteristics of a system, the dynamic loads exciting a structure are estimated by measuring induced strain fields. The accuracy of estimated loads is dependent on the placement of gages on the instrumented structure and the number of retained strain modes from strain modal analysis. A solution procedure based on the construction of a D-optimal design is implemented to determine the optimum locations and orientations of strain gages that will provide the most precise load estimates. A novel approach is proposed which makes use of model reduction technique, resulting in significant accuracy in the dynamic load estimation. Validation of the proposed approach through numerical example problems is also presented which reveals the effectiveness and robustness of the technique even in the presence of errors (noise) in strain measurements.

1. Introduction

Knowledge of loads acting on a component early in the design process is vital for design optimization and effective analysis that ensures the structural integrity of the component. Accurate estimation of the loads leads to greater confidence in numerical simulation such as finite element analysis, which significantly reduces the reliance on expensive and time consuming experimental testing. In many instances, it is possible to introduce load transducers (load cells) between the structure and the load transferring body that can directly measure the loads acting on the structure. This method of load measurement, however, suffers from certain limitations. For instance, introduction of load transducers can change the system dynamic characteristics leading to inaccurate load estimation. In some applications, the load locations may not be accessible to facilitate insertion of load transducers to measure the loads being transmitted to the structure. In several other applications, direct measurement of the excitation loads is not feasible such as aerodynamic loads, seismic excitation, engine torque pulses, fluid-flow induced forces in piping systems etc.

In many applications, it is possible to measure the response of the structure to the unknown applied loads. The response may be quantities such as displacements, accelerations, strains etc. that depend on the loads, and their measurement is

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Nomenclature

n	number of degrees of freedom of the system
e	number of elements in the FE model of the structure
c	($c \leq e$) number of elements suitable for mounting strain gages
g	($g \leq c$) number of strain gages
m	($m \leq g$) number of available/retained modes
$\dot{\{\}}$	derivative with respect to time
$[M]$	$(n \times n)$ mass matrix
$[C]$	$(n \times n)$ damping matrix
$[K]$	$(n \times n)$ stiffness matrix
$[\phi]$	$(n \times n)$ modal matrix
$\{f(t)\}$	$(n \times 1)$ load vector
$\{x(t)\}$	$(n \times 1)$ displacement vector
$\{q(t)\}$	$(n \times 1)$ mode participation factor (MPF)
$\{\varepsilon(t)\}$	$(e \times 1)$ elemental strain vector
$[\psi^{\varepsilon}]$	$(e \times n)$ modal strain matrix
$[\tilde{\psi}^{\varepsilon}]$	$(e \times m)$ truncated modal strain matrix retaining only m modes
$\{\tilde{q}(t)\}$	$(m \times 1)$ mode participation factor for retained modes
$[\tilde{\psi}^{\varepsilon}]_{cs}$	$(c \times m)$ candidate set; subset of $[\tilde{\psi}^{\varepsilon}]$
ε_{ei}	experimentally measured strain from gage i
ε_{pi}	predicted strain for gage i
$[\tilde{\tilde{\psi}}]^{\varepsilon}$	$(g \times m)$ a random subset of $[\tilde{\psi}^{\varepsilon}]_{cs}$
$\{\tilde{\tilde{q}}(t)\}$	$(m \times 1)$ approximation to $\{\tilde{q}(t)\}$
$\{\tilde{\varepsilon}(t)\}$	$(g \times 1)$ strain vector at randomly chosen locations on the structure
$[\varepsilon]_{xyz}$	(3×3) strain tensor in xyz coordinate system
$[T]$	(3×3) rotational transformation matrix
$[\varepsilon]_{x'y'z'}$	(3×3) strain tensor in $x'y'z'$ coordinate system
$[\psi^{\varepsilon}]_{opt}$	$(g \times m)$ optimum subset of $[\tilde{\psi}^{\varepsilon}]_{cs}$ determined by D-optimal design
$\{\varepsilon(t)\}_{opt}$	$(g \times 1)$ strain vector at optimum strain gage locations
r	number of boundary DOFs
b	$(r \times 1)$ boundary DOFs
i	$((n-r) \times 1)$ internal DOFs
$[I]$	$(r \times r)$ identity matrix
p	number of Craig–Bampton constrained normal modes
$[\phi]_c$	$((n-r) \times p)$ Craig–Bampton constrained modal matrix
$\{q(t)\}_p$	$(p \times 1)$ MPF of the constrained normal modes
$[0]$	$(r \times p)$ zero matrix
$[\psi]_{CB}$	$(n \times (r+p))$ Craig–Bampton transformation matrix
$[M]_{CB}$	$((r+p) \times (r+p))$ Craig–Bampton reduced mass matrix
$[C]_{CB}$	$((r+p) \times (r+p))$ Craig–Bampton reduced damping matrix
$[K]_{CB}$	$((r+p) \times (r+p))$ Craig–Bampton reduced stiffness matrix

more feasible than measuring the loads directly. A linear relationship (also called the system transfer function) between the loads to be estimated and the measured quantity can then be employed, along with the principle of superposition, to estimate the imposed loads. The instrumented structure, thus, behaves as its own load transducer. This class of problems is known as the "inverse problem".

Solving the inverse problem may seem to be a straightforward task, but unfortunately this notion is misleading because the inverse problem tends to be highly ill-conditioned. Historically, the inverse problem pertaining to load identification has been studied extensively in time, modal and frequency domains. Stevens [1] presented an excellent overview of the difficulties posed by this class of inverse problems. Kinematic response measurements using displacement transducers and accelerometers are well established and well documented by Ewins [2]. It has been noted that even very small variations (noise) in the response measurement can cause large errors in the force estimation. Desanghere [3] was one of the first researchers to study the load identification problem in frequency domain and attributed the reason for ill-conditioning to a few dominant elements in the Frequency Response Function (FRF) matrix. Okubo et al. [4] studied the influence of noise contaminating the measured response as well as the FRF on the accuracy of force estimation and found the inversion process to be highly ill-posed.

To reduce the effect of noise, Busby and Trujillo [5] cast the load estimation problem as a minimization problem of error which is defined as the difference between measured structural response and response predicted from the model. They used dynamic programming to solve this minimization problem resulting in force estimation based on a recursive reformulation of the governing equations. Hollandsworth and Busby [6] extended the previous study [5] by applying it to actual experimental measurements. One of the disadvantages of the method is that the amount of computation increases dramatically as the model order increases.

Starkey and Merrill [7] investigated the reason for the errors encountered in predicting the forces in frequency domain. They concluded that the ill-conditioned nature of the equation is due to the fact that the FRF matrix is frequently near-singular. Hansen and Starkey [8], working on a similar line [7], investigated the ill-conditioned nature of the modal model method. Their study was based on the effect of locations of accelerometer placements on a steel beam on the condition number of the modal matrix. They concluded that the condition number of the modal matrix can be improved through proper selection of the accelerometer placement and modes included in the analysis.

Carne et al. [9] proposed a technique referred to as the Sum of Weighted Acceleration Technique (SWAT) that estimates the input forces by summing the weight-scaled measured accelerations. Genaro and Rade [10] developed a technique based on identified eigen-solutions to reconstruct input forces from acceleration response. Kammer [11] used acceleration