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Efficient multi-order uncertainty computation for stochastic subspace identification

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ABSTRACT

Stochastic Subspace Identification methods have been extensively used for the modal analysis of mechanical, civil or aeronautical structures for the last ten years. So-called stabilization diagrams are used, where modal parameters are estimated at successive model orders, leading to a graphical procedure where the physical modes of the system are extracted and separated from spurious modes. Recently an uncertainty computation scheme has been derived for allowing the computation of uncertainty bounds for modal parameters at some given model order. In this paper, two problems are addressed. Firstly, a fast computation scheme is proposed reducing the computational burden of the uncertainty computation scheme by an order of magnitude in the model order compared to a direct implementation. Secondly, a new algorithm is proposed to derive efficiently the uncertainty bounds for the estimated modes at all model orders in the stabilization diagram. It is shown that this new algorithm is both computationally and memory efficient, reducing the computational burden by two orders of magnitude in the model order.

1. Introduction

Subspace-based system identification methods have proven to be efficient for the identification of linear time-invariant (LTI) systems, fitting a linear model to input/output or output only measurements taken from a system. An overview of subspace methods can be found in [1–4]. During the last decade, subspace methods found a special interest in mechanical, civil and aeronautical engineering for *modal analysis*, namely the identification of *vibration modes* (eigenvalues) and *mode shapes* (corresponding eigenvectors) of structures. Therefore, identifying an LTI system from measurements is a basic service in vibration monitoring [see e.g. 5–8]. Having done this allows in particular Finite Element Model updating and Structural Health Monitoring.

In Operational Modal Analysis, the true model order is hardly known and moreover spurious modes appear in the estimated models. Usually, an empirical multi-order estimation procedure is used, where the system is identified at multiple (over-specified) model orders in order to distinguish the true structural modes from spurious modes using the so-called stabilization diagrams [3,9,10]. There, the true structural modes are assumed to stabilize when the model order increases and thus can be separated from the spurious modes.

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Nomenclature

$\mathbb{R}$, $\mathbb{C}$	sets of real and complex numbers
$\Re$, $\Im$	real, imaginary part
$\otimes$	Kronecker product
vec	column stacking vectorization operator
†	Moore–Penrose pseudoinverse
ΔX	first order perturbation on X
$\mathcal{J}_{Y,X}$	sensitivity of vec(Y) wrt. vec(X)
$O(\cdot)$	Landau notation for complexity
A, C	system matrices
n	system order
n_m	maximal system order
n_d	number of modes
n_b	number of data blocks
r, r_0	number of sensors, reference sensors
λ_i	eigenvalue of A
ϕ_i, χ_i	right and left eigenvector of A
φ_i	mode shape
f_i, ξ_i	frequency, damping ratio
τ	sampling time step
$\mathcal{H}$	subspace matrix of size $(p+1)r \times qr_0$
$\Sigma_\mathcal{H}$	covariance of vec($\mathcal{H}$)
T	factor of estimate $\widehat{\Sigma}_\mathcal{H} = TT^T$
u_j, v_j, σ_j	left, right singular vector and value of $\mathcal{H}$
$\mathcal{O}$	observability matrix
$\mathcal{O}^\uparrow$, $\mathcal{O}^\downarrow$	$\mathcal{O}$ without last/first block row
S_1, S_2	selection matrices with $S_1\mathcal{O} = \mathcal{O}^\uparrow$, $S_2\mathcal{O} = \mathcal{O}^\downarrow$
I_a	identity matrix of size $a \times a$
$0_{a,b}$	zero matrix of size $a \times b$
$\mathcal{P}_{a,b}$	permutation, $\text{vec}(X^T) = \mathcal{P}_{a,b}\text{vec}(X)$, $X \in \mathbb{R}^{a,b}$

The estimated modal parameters are afflicted with statistical uncertainty for many reasons, e.g. finite number of data samples, undefined measurement noises, non-stationary excitations, etc. Then, the system identification algorithms do not yield the exact system matrices. The statistical uncertainty of the obtained modal parameters at a chosen model order can be computed from the uncertainty of the system matrices, which depends on the uncertainty in the data due to noise and turbulence. In [11], it has been shown how uncertainty bounds for modal parameters can be obtained in such a way. An analysis of this approach and an in depth literature review on the subject is found in [12], where also difficulties in developing confidence intervals on modal parameters from subspace identification are pointed out.

A direct and naive implementation of the uncertainty computation method in [11] is computationally taxing, especially when dealing with large sensor sets and a high model order. It has been derived for a fixed given model order and without giving implementation details. In practice, system identification results are needed at multiple model orders for the computation of the stabilization diagram. Then, redoing the uncertainty computations at several increasing model orders yields an expensive computational burden already for moderate system orders. In this paper, efficient implementations and new algorithms are proposed to solve this problem. Firstly, the algorithm in [11] is mathematically reformulated, resulting in an efficient implementation with a computational boost in one order of magnitude in the considered model order compared to the naive implementation. Secondly, a new algorithm is proposed for the computation of uncertainty bounds at multiple model orders corresponding to all modes in a stabilization diagram. It is shown how the computation of uncertainty bounds at any lower model orders can be done at a very low cost, when computations at the maximal desired model order are already done. This results in a decrease of the computational complexity of two orders of magnitude in the maximal model order. The new schemes are derived for the computation of uncertainty bounds of natural frequencies, damping ratios and mode shapes successively. The corresponding computational cost for each part of the computations of the desired modal parameters is addressed and compared to the naive implementation of the original algorithm in [11].

The paper is organized as follows. In Section 2, some preliminary modeling and the general subspace methods are given. In Section 3, the principle of the covariance computations is explained. In Section 4, notations and results of the uncertainty computations obtained in [11] are recalled and reformulated in Section 5 for a fast implementation. The computational burden of the implementations is analyzed and compared in Section 6. In Section 7 the new multi-order uncertainty computation algorithms are derived and their merits in terms of computational cost are discussed. A numerical example is given in Section 8, where the efficiency of the new algorithms is demonstrated.

2. Stochastic subspace identification (SSI)

2.1. Vibration modeling

The behavior of a vibrating structure is described by a continuous-time, time-invariant, linear dynamical system, modeled by the vector differential system

$$\begin{cases} \mathcal{M}\ddot{x}(t)+\mathcal{C}\dot{x}(t)+\mathcal{K}x(t)=\upsilon(t) \\ y(t)=Lx(t) \end{cases}, \tag{1}$$

where t denotes continuous time; $\mathcal{M}$, $\mathcal{C}$, $\mathcal{K} \in \mathbb{R}^{d\times d}$ are mass, damping, and stiffness matrices, respectively; the (high