Digital Signal Processing 39 (2015) 15–21

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

A model of active trading by using the properties of chaos

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ARTICLE INFO

Article history:
Available online 7 January 2015

Keywords:
Chaotic time series
Lyapunov exponent
Stock market return
Active trading
BIST-100 index
Shenzen index

ABSTRACT

This study introduces a research path to obtain alternative trading rules by using nonlinear dynamical analysis of stock returns. We examine the daily return data of Istanbul Stock Exchange index and Shenzhen Index B-Shares. Both stock returns series are shown to exhibit chaotic behavior and associated maximal Lyapunov exponents (LE) are computed. A new prediction method which bases on the properties of detected chaotic behavior is proposed to perform one-week out-of-sample prediction of the stock returns. Finally we develop a nonlinear model of active trading, in which traders rely only on their heterogeneous forecasts of future periods' maximum and minimum returns. The model motivates active trading under chaotic behavior.

1. Introduction

Economists have long been fascinated by the nature and sources of variations in the stock market. By the early 1970s a consensus had emerged among financial economists suggesting that stock prices could be well approximated by a random walk model and that changes in stock returns were basically unpredictable (see Fama [1], recently honored with the Nobel Prize 2013 in Economic sciences). The efficient market hypothesis (EMH) evolved in the 1960s from the random walk theory of asset prices advanced by Samuelson [2]. The securities markets were believed to be efficient in conveying information about individual stocks and about the stock market as a whole. Samuelson showed that in an information-based efficient market price changes must be unpredictable, a fact later emphasized by Sims [3]. In a recent study, Cochrane ([4]: 389) points out that based on EMH framework, any apparent predictability[2] is either statistical artifact which quickly vanishes out of sample, or cannot be used to device profitable trading strategies given the incidence of transaction costs. On the other hand, some researchers reported systematical departures from the EMH (for a detailed survey see Malkiel [5] and Cochrane [4], Ch. 20). Thanks to the discovery that less complicated nonlinear systems can follow complex and chaotic dynamics, and following the studies of Grassberger and Procaccia [6], Brock et al. [7], Wolf et al. [8] the systematical departures of market prices, i.e., stock prices, exchange rates, oil prices, interest rates, from the EMH were brought to light. The main improvement in the empirical studies comes with the studies of Rosenstein et al. [9] and Kantz [10] who reduced computational intensity and increased test power per required parameters. Thus, researchers increasingly focus on evidence of deterministic chaos in economic and financial process even with small data sets (for recent studies see [11–14]). We observe that even though the chaotic behavior of some well-known stocks are documented by various studies,[3] to the best of our knowledge there are few studies looking for the predictability of the stock returns by using Lyapunov exponents (among others see Wang et al. [11]). The first aim of our study is to add to this literature. In this context by using the properties of chaos theory we try to predict the daily return values of the Istanbul Stock Exchange (ISE-100) and the Shenzhen stock exchange (SZSE) B-shares. These two indexes play important roles respectively in the Turkish and Chinese economies, which have the highest growth rates at last decade among emerging markets. Secondly, to the best of our knowledge, the studies reporting evidence on nonlinear predictability neither examine the sources of achieved predictability nor propose a model for active trading rule. Exceptionally Brock et al., [21] and Gencay [22] focused on the relationship between returns and buy–sell signals. However, both

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[2] Throughout the study the term "predictability" refers to out-of-sample forecasting.

[3] To conserve space we document some of these studies. Scheinkman and LeBaron [15] report the existence of the nonlinearity for U.S. weekly returns on the Center for Research in Security Prices (CRSP) value-weighted index, and find rather strong evidence of chaos. For U.S. stock-market index Mayfield and Mizrach [16], Vaidyanathan and Krehbiel [17] report chaotic behavior. Peters [18] examined the S&P 500 and showed strong evidence of chaos in S&P 500 index. For the survey of chaos studies on world-wide stock returns please refer to Abhyankar et al. [19]. More recently, for the firm level rather than the properties of indices, Hagtvedt [20] reported evidence of chaos.

http://dx.doi.org/10.1016/j.dsp.2014.12.013

studies are based on the past buy and sell signals of the moving average (MA) rules, but not on chaotic dynamics. Our study also aims to fill this gap by proposing a nonlinear active trading rule which motivates active trading even under chaotic behavior of stock returns and which can be alternative to the EMH models.

Note that our study presents certain similarities with the studies of Wang et al. [11], Caglar et al. [13] and Ou and Lai [23] from various respects. Based on the algorithm proposed by Grassberger and Procaccia [6] to compute correlation dimension, Caglar et al. [13] showed that over the period from July 1987 to January 2006 the time series of Istanbul Stock Exchange (ISE) index daily returns exhibit chaotic behavior. The authors conclude that long-term prediction is not possible for daily return series of ISE ([13]: 1397). However, the authors did not compute the maximal LE.

Ou and Lai [23] demonstrated that US Dollar–Taiwan Dollar (USD–TWD) exchange rate time series have chaotic dynamics over the period from June 2007 to August 2010. To compute the maximal Lyapunov exponent, the authors implemented the methodology introduced by Wolf et al. [8]. However, it has been long understood that the algorithm proposed in [8] is unreliable for small data sets, computationally intensive and relatively difficult to implement (see Rosenstein et al. [9]: 119; Kantz [10]: 84). Moreover, in order to compute max LE by Wolf's algorithm one has to compute angular separation between neighbor points as well[4] (see Wolf et al. [8]: 295). However, we could not find that parameter in Ou and Lai [23]. Besides, the authors do not state whether they analyzed nominal exchange rate or real (effective) exchange rate. The former depends upon self-fulfilling expectations of active traders (multiple equilibria, see Cooper [24]), whereas the latter is mostly driven by the former, by the purchasing power parity (and by the trade volume).

Wang et al. [11] analyzed the complex dynamical behaviors of the daily time series, including opening quotation, closing quotation, maximum price, minimum price, total volume of the two stocks exchanged in SZSE and SHSE stock market indexes, respectively. Based on the methodology proposed by Rosenstein et al. [9] to compute max LE, the authors reported chaotic behavior of the above-listed five time series of a stock exchanged in SZSE. Then the authors proposed a "prediction method" and performed in-sample "prediction" for the total volume series of the stock which was determined to be chaotic. Even though Wang et al. [11] prefer the term "prediction", the authors' approach does not cover "out-of-sample" data, but is performed through "in-sample" data.[5] Thus the task carried by Wang et al., is neither a prediction nor a forecasting, but is an in-sample fit of the model (see Brooks [25]: 279; Peseran [26]). From this perspective our study differs from theirs. To shed light on future studies, we find it worthwhile to introduce a technical discussion on approach of Wang et al. [11]. It can be found in Section 4.

The rest of the study is organized as follows: in Section 2, basic concepts of phase space reconstruction, correlation dimension, and Lyapunov exponent are discussed. In Section 3, the phase space reconstructions, correlation dimensions and maximal Lyapunov exponents, are computed for the daily data series of ISE-100 and SZSE B. An approach for out-of-sample prediction of the daily data series is proposed in Section 4. In Section 5 we introduce a nonlinear model of active trading based on subjective trading rules. Conclusions are finally drawn in Section 6.

[4] Put differently, it should be guaranteed to be below some threshold. We consider that this is nearly impossible since the dynamical system generating exchange rates is unknown to observer and hence equations of motion.

[5] The authors' analysis on SZSE index covers the period from 31 December 1996 to 14 March 2002, but the "prediction" begins from 15 March 2001 and goes through 250 days (Wang et al. [11]: 254). This means that all information from 15 March 2001 onwards has been used when estimating model parameters.

2. Fundamentals of analysis method

Let us denote the dynamical system, $f : R^n \to R^n$, with the trajectory,

$$x_{t+1} = f(x_t) + \varepsilon_{t+1}, \quad t = 0, 1, 2, \dots, \tag{1}$$

The dynamical system itself may be assumed to be contaminated by noise, or the observed time series z_t given in Eq. (3) may be assumed to convey noise.[6] The Lyapunov exponents for such a dynamical system are measures of the average rate of divergence or convergence of a typical trajectory or orbit. The trajectory is also written in terms of the iterates of f. For an n-dimensional system as given above, there are n exponents which are customarily ranked from largest to smallest:

$$\lambda_1 \ge \lambda_2 \ge \dots \ge \lambda_n \tag{2}$$

One rarely has the advantage of observing the state of the system at any period t, x_t, and that the actual functional form, f, that generates the dynamics. The model that is widely used is the following: associated with the dynamical system in Eq. (1) there is a measurement function $h : R^n \to R$ which generates the time series,

$$z_t = h(x_t) \tag{3}$$

It is assumed that all that is available to observer is the sequence $\{z_t\}$.

Assume that the target system is a dynamical system as given in Eq. (1), and that the observed time series is obtained through a measurement function as given in Eq. (3). Then, following Takens' theorem [27], the reconstructed trajectory is an embedding of the original trajectory when the m value is sufficiently large. In order that such reconstruction achieves embedding, the dimension m should satisfy $m \ge 2n + 1$. However, this is a sufficient condition and upper-worst case. Depending on the data, embedding can be established even when m is less than $2n + 1$ (Gencay and Dechert [28]). In the embedding method, there are two parameters: embedding dimension and time delay. Abarbanel [29] suggests how to select m and d. From now on the time delay d, is taken to be equal to 1, which corresponds to our observation interval on time domain.

According to Takens [27], from observed time series $\{z_t\}$, one can generate the data vector

$$y_i = (z_i, z_{i+d}, \dots, z_{i+(m-1).d}) \quad \text{for all } i \in \big(N - (m-1).d\big) \tag{4}$$

where N is the length of the observed sequence $\{z_t\}$. This vector indicates a point of m-dimensional reconstructed phase space R^m. Thus a point on the orbit is obtained as given in Eq. (4).

Grassberger and Procaccia [6] introduced a method for calculating correlation dimension which is widely used for characterizing strange attractors. The correlation dimension is defined as follows.

Let r denote critical distance. For a given m, the correlation integral is defined as

$$C_K(r,m) = \frac{2}{K(K-1)} \sum_{i,j=1\ (i \ne j)}^{K} \theta\big(r - |y_i - y_j|\big) \tag{5}$$

where K is the cardinality of the dataset, θ is Heaviside function, if $v \le 0$, $\theta(v) = 0$; otherwise $\theta(v) = 1$, for a phase point y_i. If r is too small, then $C_K(r,m) = 0$. If it is taken too high, then $C_K(r,m) = 1$.

[6] Kantz's algorithm allows us to make this assumption, see [9] and [10].