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A Maximum Entropy inspired model for the convolutional noise PDF

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ABSTRACT

In this paper we consider a blind adaptive deconvolution problem in which we observe the output of an unknown linear system (channel) from which we want to recover its input using an adaptive blind equalizer (adaptive linear filter). Since the channel coefficients are unknown, the optimal equalizer's coefficients are also unknown. Thus, the equalizer's coefficients used in the deconvolution process are only approximated values leading to an error signal in addition to the source signal at the output of the deconvolutional process. We define this error signal throughout the paper as the convolutional noise. It is well known that the convolutional noise probability density function (pdf) is not a Gaussian pdf at the early stages of the deconvolutional process and only at the latter stages of the deconvolutional process the convolutional noise pdf tends to be approximately Gaussian. Despite this knowledge, the convolutional noise pdf was modeled up to recently as a Gaussian pdf because it simplifies the Bayesian calculations when carrying out the conditional expectation of the source input given the equalized or deconvolutional output and since no other model was suggested for it. Recently, a new model was suggested by the same author for the convolutional noise pdf based on the Edgeworth expansion series. This new model leads to improved deconvolution performance for the 16 Quadrature Amplitude Modulation (QAM) input and for a signal to noise ratio (SNR) of 30 dB. Thus, the question that arose here was whether we may find another model for the convolutional noise pdf that will also lead the system with improved deconvolutional performance compared to the case when the Gaussian model is applied for the convolutional noise pdf. In this paper, we propose a new model for the convolutional noise pdf inspired by the Maximum Entropy density approximation technique. We derive the relevant Lagrange multipliers and obtain as a by-product new closed-form approximated expressions for the conditional expectation and mean square error (MSE). Simulation results indicate that improved system performance is obtained from the residual ISI point of view for the 16QAM input case with our new proposed model for the convolutional noise pdf compared to the case when the Gaussian model or Edgeworth expansion series are applied for the convolutional noise pdf. For two other chosen input sources, a faster convergence rate is observed with the algorithm using our new proposed model for the convolutional noise pdf compared to the Maximum Entropy and Godard's algorithm.

1. Introduction

In this paper, we deal with the convolutional noise arising at the output from a blind deconvolutional process. A blind deconvolution process arises in many applications such as seismology, underwater acoustic, image restoration and digital communication [1]. Let us consider for a moment the digital communication case. During transmission, a source signal undergoes a convolutive distortion between its symbols and the channel impulse response. This distortion is referred to as intersymbol interference (ISI) [2]. Thus, a blind adaptive filter is used to remove the convolutive effect of the system to produce the source signal [2]. This process is called blind deconvolution. Since the updated coefficients used in the blind adaptive filter are not the ideal values, a noise named as "convolutional noise" is observed at the output of the deconvolution process in addition to the source signal.

According to [1] and [3], in the early stages of the iterative deconvolution process, the ISI is typically large with the result that the data sequence and the convolutional noise sequence are strongly correlated and the convolutional noise sequence is more uniform than Gaussian [4]. In the latter stages of the deconvolution process where the blind adaptive filter has reached the convergence state, the convolutional noise pdf is approximately Gaussian [3]. Despite of this knowledge, the convolutional noise pdf was modeled up to recently as Gaussian [1,3,5–11]. In [5], the Gaussian pdf model was used to obtain the residual ISI obtained by blind

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adaptive equalizers while in [1,3,6–11] the Gaussian pdf model was applied to obtain the conditional expectation of the source signal given the equalized output via Bayes rules. There is no doubt that applying the Gaussian pdf model for the convolution noise pdf simplifies the calculations but at the same time leads to suboptimum solutions from the residual ISI point of view [1,3].

In the literature we may find several works [12–16] dealing with pdfs applicable for the non-Gaussian case but encompasses also the Gaussian model. The multiple-user interference (MUI) in time hopped impulse-radio ultrawide bandwidth (UWB) systems is impulse-like and poorly approximated by a Gaussian distribution [12]. Several alternative distributions for approximating the MUI process and the MUI-plus-noise process in UWB systems are motivated and compared in [12]. In [16], the approach of using two pdfs is adopted to approximate the distribution of the multiuser interference plus noise. Specifically, a generalized Gaussian pdf is used in [16] to approximate the distribution of the total disturbance when the received pulse is collided and a Gaussian pdf is applied to approximate the distribution of the total disturbance when the received pulse is not collided. In [14], a new detector is proposed for amplify-and-forward (AF) relaying system when communicating with the assistance of relays where the probability density function is estimated with the help of the kernel density technique and a generalized Gaussian kernel is proposed that provides more flexibility and encompasses Gaussian and uniform kernels as special cases. According to [15], complex elliptically symmetric (CES) distributions have been widely used in various engineering applications for which non-Gaussian models are needed. CES distributions constitute a wide-class of distributions that include, among others, the complex normal, complex t-distribution, K-distribution, complex generalized Gaussian distribution and the class of compound-Gaussian distributions as special cases. In [15], circular CES distributions are surveyed. The generalized Gaussian distribution (GGD) provides a flexible and suitable tool for data modeling and simulation [13]. In [13] a thorough presentation of the complex-valued GGD is given and denoted as CGGD. The CGGD adapts to a large family of bivariate symmetric distributions, from super-Gaussian to sub-Gaussian including specific densities such as Laplacian and Gaussian distributions [13].

Up to recently, the Gaussian model was the only option on the table for modeling the convolutional noise pdf. Recently [17], a new model was suggested for the convolutional noise pdf based on the Edgeworth expansion series. This new model has shown to lead to improved deconvolution performance for the 16QAM input and SNR $= 30$ dB case. Thus, the question that arose here was whether it is possible to find another model for the convolutional noise pdf that will also lead the system with improved deconvolutional performance compared to the case when the Gaussian model is used for the convolutional noise pdf.

In this paper, we propose a new model for the convolutional noise pdf inspired by the Maximum Entropy density approximation technique. We derive the relevant Lagrange multipliers and obtain as a by product, new closed-form approximated expressions for the conditional expectation and MSE. Simulation results indicate that improved system performance is obtained for the 16QAM input case with our new proposed model for the convolutional noise pdf compared to the case when the Gaussian model or Edgeworth expansion series are applied for the convolutional noise pdf.

This paper is organized as follows: after having described the system under consideration in Section 2 we introduce our new model for the convolutional noise pdf in Section 3. In this section, we also obtain new closed-form approximated expressions for the conditional expectation and MSE as well as the needed Lagrange multipliers. Simulation results are given in Section 4 and Section 5 is our conclusion.

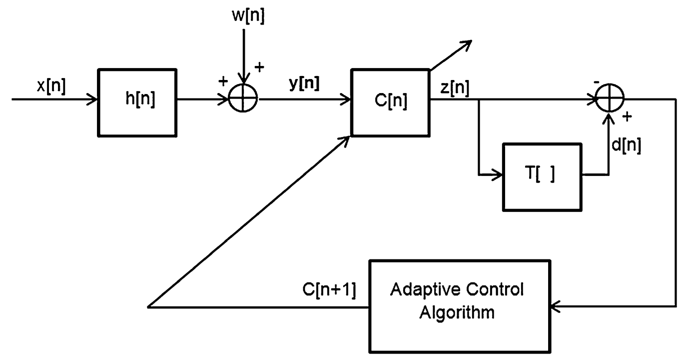

Fig. 1. A block diagram for baseband communication transmission.

2. System description

The system under consideration is the same system used in [1] and is illustrated again in Fig. 1.

The following assumptions have been taken following [1]:

1. The input sequence $x[n]$ consists of zero mean, real or complex (where the real and imaginary parts are independent) random variables with unknown even symmetric probability distribution function.
2. The unknown channel $h[n]$ is a possibly non-minimum phase linear time-invariant filter where the zeros lie sufficiently far from the unit circle.
3. The equalizer $c[n]$ is a tap delay line.
4. The noise $w[n]$ is an additive Gaussian white noise (AWGN).
5. The function $T[\cdot]$ is a memoryless nonlinear function (called “the nonlinearity”) which satisfies: $T[z_1 + jz_2] = T[z_1] + jT[z_2]$ where z_1 and z_2 are the real and imaginary parts of the equalized output respectively.

The input sequence $x[n]$ is transmitted through the channel $h[n]$ and is corrupted with noise $w[n]$. Therefore, the equalizer's input sequence $y[n]$ may be written as:

$$y[n] = x[n] * h[n] + w[n] \tag{1}$$

where ‘*’ denotes the convolution operation. In the ideal case, the equalized output could be written as [8]:

$$z[n] = x[n - D]e^{j\theta} \tag{2}$$

where D is a constant delay and θ is a constant phase shift. Therefore, for the ideal case we have:

$$c[n] * h[n] = \delta[n - D]e^{j\theta} \tag{3}$$

where δ is the Kronecker delta function. In this paper we assume that $D = 0$ and $\theta = 0$, since D does not affect the reconstruction of the original input sequence $x[n]$ and θ can be removed by a decision device [1,8]. Let $c_g(n)$ be the initial guess for the unknown $c[n]$. Thus, we may write [8]:

$$\tilde{s}[n] = c_g[n] * h[n] = \delta[n] + \xi[n] \tag{4}$$

where $\xi[n]$ stands for the difference between the ideal value $c[n]$ and the initial guess $c_g[n]$. According to [3], at the latter stages of the deconvolutional process, $\xi[n]$ may be considered as a long and oscillatory wave and if $\xi[n]$ is long enough, the central limit theorem makes a Gaussian model for the convolutional noise $(\xi[n] * x[n])$ to be plausible. Convolving $c_g[n]$ with the received sequence $y[n]$, using (1) and (4) we obtain:

$$z[n] = y[n] * c_g[n] = x[n] + p[n] + \tilde{w}[n] \tag{5}$$