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The effect of prediction error correlation on optimal sensor placement in structural dynamics

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A B S T R A C T

The problem of estimating the optimal sensor locations for parameter estimation in structural dynamics is re-visited. The effect of spatially correlated prediction errors on the optimal sensor placement is investigated. The information entropy is used as a performance measure of the sensor configuration. The optimal sensor location is formulated as an optimization problem involving discrete-valued variables, which is solved using computationally efficient sequential sensor placement algorithms. Asymptotic estimates for the information entropy are used to develop useful properties that provide insight into the dependence of the information entropy on the number and location of sensors. A theoretical analysis shows that the spatial correlation length of the prediction errors controls the minimum distance between the sensors and should be taken into account when designing optimal sensor locations with potential sensor distances up to the order of the characteristic length of the dynamic problem considered. Implementation issues for modal identification and structural-related model parameter estimation are addressed. Theoretical and computational developments are illustrated by designing the optimal sensor configurations for a continuous beam model, a discrete chain-like stiffness–mass model and a finite element model of a footbridge in Wetteren (Belgium). Results point out the crucial effect the spatial correlation of the prediction errors have on the design of optimal sensor locations for structural dynamics applications, revealing simultaneously potential inadequacies of spatially uncorrelated prediction errors models.

1. Introduction

The problem of parameter estimation of structural models using measured dynamic data is important in modal identification, structural model updating, structural health monitoring and structural control. The estimate of the parameter values involves uncertainties that are due to limitations of the mathematical models used to represent the behavior of the real structure, the presence of measurement error in the data and insufficient excitation and response bandwidth. In particular, the quality of information that can be extracted from the data for estimating the model parameters depends on the number and location of sensors in the structure as well as on the type and size of model and measurement error. The objective in an experimental design is to make a cost-effective selection of the optimal number

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and location of sensors such that the resulting measured data are most informative for estimating the parameters of a mathematical model of the structure.

Information theory based approaches [1–8] have been developed to provide rational solutions to several issues encountered in the problem of selecting the optimal sensor configuration for modal identification and structural parameter estimation. The optimal sensor configuration is selected as the one that maximizes some norm (determinant or trace) of the Fisher information matrix (FIM) [9]. In other studies [10,11], the optimal sensor configuration has been chosen as the one that minimizes the expected Bayesian loss function involving the trace of the inverse of the FIM. A Bayesian framework to optimal sensor location for structural health monitoring (SHM) has also been introduced in [12]. The optimal configuration is chosen to optimize (maximize or minimize) a Bayesian risk-based performance metric related to the probability of damage detection or false alarm of all regions of the structure. A probabilistic approach to optimal sensor location for SHM was proposed in [13], utilizing a priori knowledge of probable damage locations and severities in the structure. The weights of an artificial neural network, trained to detect damage, were used to generate a probability distribution function that is sampled to determine the optimal sensor locations.

The information entropy, measuring the uncertainty in the model parameter estimates, was also introduced [14] for designing optimal sensor configurations. It was shown [15] that, asymptotically for very large number of data, the information entropy depends on the determinant of the FIM, justifying the use of the determinant instead of the trace or other scalar measures of FIM in previous approaches. The information entropy has been applied to design optimal sensor locations for parameter estimation using ambient vibrations [16], model class selection [17] for damage detection, as well as to design the optimal excitation characteristics (e.g. amplitude and frequency content) for the identification of linear and strongly nonlinear models [18].

The optimal sensor location problem is formulated as a single-objective optimization problem involving discrete-valued variables. Computational efficient algorithms for solving the discrete-valued minimization problem have been proposed. Udwadia [6] demonstrated that using the trace of the FIM is computationally very attractive since the solution of the underlined discrete optimisation problem is straightforward. However, for other more popular scalar measures of uncertainties such as the determinant of the FIM or the information entropy, an exhaustive search over all possible sensor configurations is required to obtain the exact optimal sensor configuration. This approach is computationally prohibitive even for structures with a relatively small number of degrees of freedom (DOF). Heuristic optimization tools have also been developed as effective alternatives for efficiently solving these discrete optimization problems involving discrete-valued variables. In the modal identification case, efficient iterative algorithms [19,20] were proposed for sensor placement using the effective independence method. Exploiting theoretical asymptotic results on information entropy and FIM, two computationally efficient heuristic algorithms, the forward sequential sensor placement (FSSP) and the backward sequential sensor placement (BSSP) [15,17] were proposed. These algorithms construct sensor configurations for physical model parameter estimation, corresponding to information entropy values very close to lower or upper bounds of the information entropy. The numerical results indicated that the proposed heuristic algorithms provide sub-optimal sensor configurations that can be extremely good approximations of the optimal sensor configuration [15,17]. Moreover, these heuristic algorithms are very simple to implement in software and computationally very efficient. Alternatively, Genetic Algorithms [21–24] are intelligent techniques that will yield optimal solutions and in this sense can be used whenever it is deemed necessary to complement the heuristic algorithms in an effort to improve estimates.

In most information theory-based methods the effect of spatially correlated prediction errors and its importance was not adequately explored. The present study provides insight into the effect of spatially correlated prediction errors on the design of the optimal sensor locations. The information entropy is used as the performance measure of a sensor configuration. The information entropy is built from the parameter uncertainty identified by applying a Bayesian identification framework. The optimal sensor location problem is formulated as a single-objective optimization problem involving discrete-valued variables. The effectiveness of available heuristic algorithms BSSP and FSSP, known to be computationally very efficient and accurate for uncorrelated prediction errors, is explored.

Asymptotic approximations, valid for large number of data, available for the information entropy [15] for the case of uncorrelated prediction errors are extended to account for the case of spatially correlated prediction errors. Useful theoretical results are derived that show that the lower and upper bounds of the asymptotic estimate of the information entropy, corresponding, respectively, to the optimal and worst sensor configuration, are a decreasing function of the number of sensors. In addition, it is shown that for up to the characteristic length of the highest contributing mode of the structure, the spatial correlation between prediction errors forces the minimum distance between sensors to be of the order of the prediction error correlation length. Consequently, sensor placement becomes independent of the mesh size of the finite element models used for structural dynamics simulations. For distances between two sensors higher than the characteristic length, the sensor locations are affected also by the spatial variability of the response sensitivities computed by a nominal structural model. Implementation in structural dynamics is concentrated on the design of optimal sensor location for (a) modal identification and (b) estimation of structural model (e.g. finite element) parameters. Theoretical and computational developments are demonstrated by designing the optimal sensor configurations for a simply supported continuous beam model, a discrete chain-like stiffness–mass model and a finite element model of a footbridge in Wetteren (Belgium). It is illustrated that the extent of the spatial correlation of the prediction errors has an important effect on the optimal sensor locations. In addition, inadequacies of the spatially uncorrelated prediction error models are emphasized.