Mechanical Systems and Signal Processing 28 (2012) 387–398

Contents lists available at SciVerse ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Finite element model updating with positive definiteness and no spill-over

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ARTICLE INFO

Article history:
Received 21 January 2011
Received in revised form
2 November 2011
Accepted 3 November 2011
Available online 25 November 2011

Keywords:
Model updating
Partially prescribed spectral information
Inverse eigenvalue problem
Optimal matrix approximation

ABSTRACT

Updating a finite element model to match measured spectral information has been a important task for engineers. In the process, it is desirable to match only the measured spectral information without tampering with the other unmeasured and unknown eigeninformation in the original model and to maintain positive definiteness (semidefiniteness) in the coefficient matrices. In this paper, we present a new direct method for the finite element model updating. By constructing a parametric symmetric low-rank correction form, the method can preserve both no spill-over and positive definiteness (semidefiniteness) of the mass and stiffness matrices. Using the parametric form, necessary and sufficient conditions under which this problem is solvable are obtained and a minimum modification is given explicitly. Numerical examples show that the method is efficient.

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1. Introduction

Obtaining highly accurate analytical structural models is necessary for analyzing and predicting the dynamic performance of complex structures during analysis and design. By the finite element technique, vibrating phenomenon of a mechanical or civil structure may be modeled by a second-order ordinary differential system

$$M_a\ddot{q}(t)+K_aq(t)=f(t), \tag{1}$$

where $f(t)\in\mathbb{R}^n$ varies in time t, $M_a,K_a\in\mathbb{R}^{n\times n}$ are the analytical mass and stiffness matrices, respectively, and n stands for the number of degrees of freedom. In general, M_a is symmetric and positive definite, denoted by $M_a>0$, and K_a is symmetric and positive semidefinite, denoted by $K_a\geq 0$. Eq. (1) is usually known as the finite element model. For the sake of convenience, we will denote the model simply by $\{M_a,K_a\}$. It is well known that if $q(t)=xe^{i\omega t}$ is a fundamental solution of (1), then the natural frequency ω and the mode shape (eigenvector) x must solve the following generalized eigenvalue problem:

$$K_ax=\lambda M_ax,$$

where $\lambda=\omega^2$ is called the eigenvalue. A variety of numerical methods for the generalized eigenvalue problem can be found in some books (see, for example, [1,2]). Owing to the complexity of the structure, however, the finite element model is an approximate discrete analytical model of the continuous structure. Natural frequencies and mode shapes of the analytical model $\{M_a,K_a\}$ do not match very well with experimentally measured frequencies and mode shapes obtained from a real-life vibrating test. Thus, updating the existing dynamic model on the basis of modal test data is very important

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doi:10.1016/j.ymssp.2011.11.002

for predicting actual behavior of the structure precisely via the structural dynamic model. The updated model may be considered a better representation of the actual structure than the original finite element model, and can be used with more confidence to analyze and predict the dynamic responses of the structure.

Over the past years, finite element model updating problem has received considerable discussions. Various methods have been developed for correcting analytical models to predict test results more closely. These methods can be broadly classified into five classes: optimal matrix updates or direct updating methods [3–12], sensitivity methods [13–15], eigenstructure assignment techniques [16–19], minimum-rank perturbation methods [20,21] and frequency response function-based model updating techniques [22–26], and have been widely used and successfully applied to the model updating for a variety of structures. A detailed discussion of every approach is beyond the scope of the paper. Interested readers are referred to survey papers [27,28]. A detailed theoretical analysis of model updating techniques can be found in the seminal book [29].

This paper concentrates on the direct updating methods based on measured modal data. These methods seek a refined analytical model whose modal properties are in agreement with those from the experimental modal survey of the structure, so that the updated matrices are those closest to the initial analytical model $\{M_a,K_a\}$. For example, Baruch and Bar-Itzhack [3,4] obtained a closed-form solution of the updated stiffness matrix by using Lagrange multipliers for minimizing the changes in the stiffness matrix to satisfy specified constraints, assuming that the mass matrix is exact. Baruch [5] and Berman and Nagy [6] developed the analytical model improvement procedures to update the mass and stiffness matrices alternately. Wei [7,8] presented the formulations to correct both the mass and stiffness matrices based on constrained minimization theory. Applying the QR-factorization of measured modes and the best approximation theory, Dai [9] proposed a numerical method for correcting both the mass and stiffness matrices simultaneously. The constraints imposed are the orthogonality, the dynamic equation and the symmetry of the matrices to be updated. However, the sparsity pattern of the original analytical model $\{M_a,K_a\}$ may be destroyed. In order to preserve the original stiffness matrix pattern, Kabe [30], Caesar and Peter [31], Kammer [32], Smith and Beattie [33,34], Halevi and Bucher [35], and Sako and Kabe [36] developed some algorithms preserving the connectivity of the structural model. These are analogous methods that, under certain conditions, are mathematically equivalent to the stiffness matrix adjustment method [30]. However, these methods do not preserve that the updated stiffness matrix is positive semidefinite. Another concern is that these methods cannot guarantee that extra, spurious modes are not introduced into the range of the frequency range of interest [29]. The challenge, known as the no spill-over phenomenon in the engineering literature, is that in updating an existing model it is often desirable that the current vibration parameters not related to the newly measured parameters should remain invariant. Recently, assuming that the mass matrix is not updated, Carvalho et al. [37] proposed a direct method for undamped model updating with no spill-over. Chu et al. [38–40] considered damped model updating with no spill-over. In this paper, to preserve both no spill-over and positive definiteness of the mass and stiffness matrices, we will consider the problem of updating the existing analytical model so that the updated model has the following properties:

- The measured eigenvalues and eigenvectors are also the eigenvalues and eigenvectors of the updated model.
- The unmeasured eigenvalues and eigenvectors remain unchanged.
- The exploitable properties such as symmetry, definiteness of the original model are preserved.
- The difference between the updated model and the original model is minimal.

Let $\{\lambda_1,\ldots,\lambda_p;\lambda_{p+1},\ldots,\lambda_n\}$ and $\{x_1,\ldots,x_p;x_{p+1},\ldots,x_n\}$ be the n eigenvalues and eigenvectors of the analytical model $\{M_a,K_a\}$, and let $\{\mu_1,\ldots,\mu_p\}$ and $\{y_1,\ldots,y_p\}$ be a set of $p\ (p \ll n)$ eigenvalues and eigenvectors measured from an experimental structure. Mathematically, the model updating problem may be formulated as follows.

Problem MUP: Given an analytical model $\{M_a,K_a\}$, a set of its associated eigenpairs $(\lambda_i,x_i)\ (i=1,\ldots,p)$ with $p \ll n$, and another set of measured eigenpairs $(\mu_i,y_i)\ (i=1,\ldots,p)$ from an experimental or a real-life structure, update the analytical model $\{M_a,K_a\}$ to $\{M,K\}$ of the same structure such that:

(1) $M=M^T=M_a+\Delta M>0,\ K=K^T=K_a+\Delta K \geq 0$.
(2) The subset $(\lambda_i,x_i)\ (i=1,\ldots,p)$ is replaced by $(\mu_i,y_i)\ (i=1,\ldots,p)$ as p eigenpairs of the updated model $\{M,K\}$.
(3) The remaining (unknown) $n-p$ eigenpairs of the updated model $\{M,K\}$ are the same as those of the original model $\{M_a,\ K_a\}$.

Throughout this paper, the following notations will be used. Let

$$\Lambda_1 = diag\{\lambda_1,\ldots,\lambda_p\},\quad \Lambda_2 = diag\{\lambda_{p+1},\ldots,\lambda_n\},\quad \Sigma_1 = diag\{\mu_1,\ldots,\mu_p\},$$

$$X_1=[x_1,\ldots,x_p],\quad X_2=[x_{p+1},\ldots,x_n],\quad X=[X_1,X_2],\quad Y_1=[y_1,\ldots,y_p].$$

$\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. A^T stands for the transpose of a real matrix A. If a matrix A is nonsingular, then $A^{-T} \triangleq (A^{-1})^T$. I_n represents the identity matrix of size n. For $A,B\in\mathbb{R}^{m\times n}$, an inner product in $\mathbb{R}^{m\times n}$ is defined by $(A,B)=trace(B^TA)$, then $\mathbb{R}^{m\times n}$ is a Hilbert space. The matrix norm $\|.\|$ induced by the inner product is the Frobenius norm.

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