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Enhanced meta-modelling technique for analysis of mode crossing, mode veering and mode coalescence in structural dynamics

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ABSTRACT

In this paper, an enhanced meta-modelling procedure for the approximation of structural eigenfrequencies and eigenvectors is introduced. The procedure allows for correct prediction of the modal parameters in case of mode crossing, veering, and coalescence phenomena that can be observed when variations of the structural parameters occur. The procedure overcomes the erroneous approximation of these phenomena which results from a direct approximation of the modal parameters. The methodology is based on the response surface approximation of the structural matrices and on the concept of modal reduction. A comparison with a direct response surface approximation of the eigenfrequencies shows a considerable improvement in the accuracy as it is presented for a finite element frame structure.

1. Introduction

Meta-models [1] are particularly beneficial within the design phase, where the designer is analysing various features of the product for differing values of the structural parameters, aiming to provide a prototype of the final product. Various types of numerical analysis and experimental testing are performed in order to improve the quality of the component and to achieve industrial standards. The sensitivity [2,3] of structural properties of the component with respect to specific parameters and parameter combinations is especially of interest. The influence of inherent variability of structural parameters needs to be investigated for designing a robust product. In context of uncertainty quantification in structural dynamics [4], meta-models provide a versatile tool for fast approximation of structural response quantities such as displacements and velocities or structural properties such as eigenfrequencies and mode shapes as a function of structural parameters varying with respect to a reference configuration. The utilisation of response surfaces provides an approximate solution based on finite element results obtained from a planned design of experiments (DOE) and avoids a repeated solution of the full finite element model which may be computationally demanding.

A large number of different meta-modelling procedures are known, such as, for instance, linear regression [1], polynomial interpolation [5], kriging [6], radial basis functions [7], etc. Each of these procedures has some areas of application for which it is especially suitable to be utilised. A descriptive and comparative assessment of meta-modelling techniques can be found in many references (see e.g. [8,9]). All methods have in common that their range of application is limited by the number of design variables; i.e., these meta-modelling techniques are of restricted applicability due to the curse of dimensionality. However, for many problems it can be shown that there is only a dependence on few (i.e. less than 20–25) parameters. These parameters can

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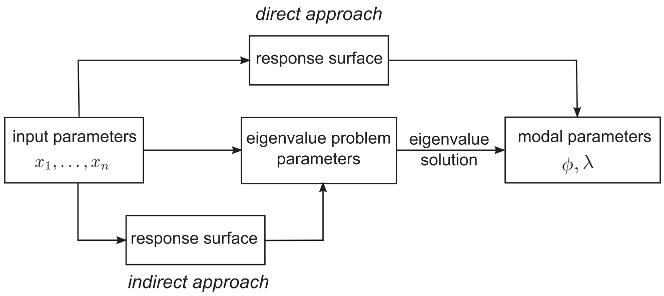

Fig. 1. Direct and indirect modal meta-modelling.

be identified by using global or local sensitivities or by engineering reasoning and expertise. In such cases the response surface methodology or other approximations are advantageous, since they may provide very accurate results at a reduced numerical cost.

In this paper linear regression with polynomial functions is proposed for approximating modal parameters of a structure, creating a so-called *modal meta-model*. Modal meta-models have been used by various authors to obtain fast predictions of the modal behaviour of a system. For example, polynomial functions have been utilised to approximate eigenvalues of civil structures [10,11] or to approximate eigenvectors of a car component [12,13]. In Refs. [14,15] the modal parameters have been represented as a linear combination of the orthogonal basis of a reference system. The modal mass and stiffness were approximated in [16] with polynomial regression. Modal meta-models have also been adopted in the field of vibro-acoustics [17]. All these investigations have in common that a direct approximation of the considered modal parameters is performed. This means that a direct relationship between system variables and modal parameters is established as shown in Fig. 1. The limitations of the applicability of direct modal meta-modelling have been assessed in recent investigations [13]. The main difficulties arise due to the fact that it is not fully clear how to describe the variation of mode shapes and eigenfrequencies when an undamped structure exhibits mode degeneration, namely mode crossing, veering and coalescence. For such cases it is difficult to compare structural modes with the modes obtained from a nominal or reference solution.

Many references (see e.g. [18,19]) deal with the topic of eigenvalue and eigenvector sensitivities. Other approaches deal with reanalysis to evaluate the eigenvalues and eigenvectors of modified structures [20] or by interacting iteratively to the eigenvalue problem solver [21]. These methodologies are only applicable within a small neighbouring range with respect to the reference solution where the modal properties are varying linearly. This paper aims to provide a more general representation of the evolution of the structural modes of undamped systems within a moderately large input parameter space via an indirect meta-modelling technique (see Fig. 1). In the indirect modal meta-modelling the objective of the approximation is not the modal parameters resulting from the solution of the eigenvalue problem as in the direct approach. Instead, quantities involved in the structural eigenvalue problem are approximated. The approach proposed in this paper is based on the approximation of the assembled structural matrices combined with the concept of modal reduction for predicting the evolution of the modal parameters. It will be demonstrated that in the presence of the aforementioned mode degeneration an indirect meta-modelling methodology performs much better than any direct meta-modelling. The occurrence of crossing or veering modes of mechanical components or structures originates from parameter changes and can be proved experimentally and numerically (see e.g. [22,12]). For various applications, such as design optimisation or uncertainty quantification, it is important to consider these phenomena. Then, so-called mode-tracking procedures (see e.g. [23]) provide a possibility to treat correctly the involved modes. Similarly, mode crossing plays an important role in aeroelasticity. Mainly, the mode variation due to the dependence on the fluid interaction or resonance effects needs to be considered. Examples are the design of wind turbines [24] and stability of airfoils [25]. This paper focusses on the application of the presented procedure to structural problems.

The paper is structured as follows. In Section 2 important issues and limitations concerning a direct approach for modal meta-modelling are discussed. In particular, Section 2.1 describes briefly mode crossing, veering and coalescence, while in Section 2.2 the effect of these phenomena on direct modal meta-modelling are pointed out. The poor quality of predicted modal parameters in the presence of mode crossing and mode veering gives the motivation for an indirect approximation method presented in Section 3. The methodology allows one to obtain an improved approximation of the evolution of eigenfrequencies and eigenvectors in the parameter space. The concept of modal reduction is described in Section 3.1, the approximation of the mass and stiffness matrices using meta-models is explained in Section 3.2 and an overview of the modal meta-modelling procedure is given in Section 3.3. Finally, a numerical example illustrates the advantage of the proposed solution in terms of accuracy when compared to conventional direct response surface approaches.

2. Motivation

2.1. Mode crossing, veering and coalescence

The dynamical behaviour of undamped structures subjected to structural parameter modifications is characterised by degenerative phenomena [26]. Mode swapping, veering and coalescence are mode degenerations which may greatly affect