Mechanical Systems and Signal Processing 25 (2011) 2399–2412

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/jnlabr/ymssp

Model selection in finite element model updating using the Bayesian evidence statistic

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ARTICLE INFO

Article history:
Received 16 August 2009
Received in revised form
25 March 2011
Accepted 2 April 2011
Available online 4 May 2011

Keywords:
Finite element model updating
Nested sampling
Model selection
Bayesian evidence
Monte Carlo

ABSTRACT

This paper considers the problem of finite element model (FEM) updating in the context of model selection. The FEM updating problem arises from the need to update the initial FE model that does not match the measured real system outputs. This inverse system identification-problem is made even more complex by the uncertainties in modeling some of the structural parameters. Such uncertainty often results in a number of competing forms of FE models being proposed which leads to lack of consensus in the field. A model can be formulated in a number of ways; by the number, the location and the form of the updating parameters. We propose the use of a Bayesian evidence statistic to help decide on the best model from any given set of models. This statistic uses the recently developed stochastic nested sampling algorithm whose by-product is the posterior samples of the updated model parameters. Two examples of real structures are each modeled by a number of competing finite element models. The individual model evidences are compared using the Bayes factor, which is the ratio of evidences. Jeffrey's scale is then used to determine the significance of the model differences obtained through the Bayes factor.

1. Introduction

System identification [17] forms an important stage of many scientific modeling problems and is mainly concerned with the derivation of mathematical models of a system from its measured dynamics. The results from the system identification can then be used to understand and predict the system responses in future designs or different environments. Therefore the analyst is interested in the accuracy, confidence range, and more critically the correctness of the assumed mathematical model.

In this paper, the systems considered are structural and the model domain is that of finite element models (FEMs). In this context, these models are used to approximate the structural dynamics of systems such as train chassis, aircraft fuselages, bicycle frames, or civil structures. It is often the case that the finite element predictions do not match the measured structure's dynamic response [10,12]. This inconsistency may be due to the following: the form of the FE model, the identity and magnitude of the uncertain parameters, the noise and/or errors in the measurements. Furthermore the

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doi:10.1016/j.ymssp.2011.04.001

measured data is incomplete due to the impracticality of capturing the full dynamics of the structure at every degree of freedom and over the complete frequency range, and this renders the problem ill-conditioned [9,10,12]. Consensus on this inverse problem is made more difficult by the fact that a multitude of mathematical models of the structure can be developed from engineering judgment. Moreover, these models can have varying levels of complexity, which leads to non-unique solutions for a particular modeled system. This situation makes the identification of the best FEM of the system of primary importance.

A number of pertinent questions arise in system identification: (1) Which aspects of the models do we need to update [14,24]? (2) How are we to update these models? [6,13,15,18,22,23] (3) Is the updated model the best one [16]? In this paper we propose approaching the finite element model updating problem by answering the third question. This can be recast as; what *evidence* do we have that our updated model is the correct one given that a number of plausible models of the real system can be generated from answering the first two questions? Question one is normally addressed from engineering judgment, often complemented by some form of parameter sensitivity measures/studies [9,10,12,24,33]. Question two is guided by the relevance of current available methods/approaches in the field. The model evidence approach proposed in this paper has received little attention in the FEM updating field and we believe it goes to the core of the model updating problem (FEMUP). By addressing the third question, we can get a better understanding of the given structural model design, the qualitative significance of the difference between proposed models and which model(s) is (are) worth considering for further analysis. This evidence measure, we argue, ought to be an essential and necessary measure to establish before any model updating problem is settled. We thus propose that the FEMUP should be approached from a model selection perspective. This is perhaps best approached using Bayesian inference, which allows one to deal with models in an intuitive and systematic way [3,4,17,19,20].

In the next section, we formally present the finite element model background. In Section 3, we introduce parameter estimation and model selection in the Bayesian framework. The Bayesian definition of model evidence is also presented. Section 4 introduces the evidence calculation algorithm. Section 5 presents two example implementations of the evidence concept: firstly on a simple unsymmetrical H-beam and secondly on a more complex Garteur SM-AG19 structure. At the end of each example we discuss the findings and Section 6 concludes the paper.

2. Finite element background

In finite element modeling, dynamic structures are analyzed by discretizing the structure into constituent elements. When assembled these elements constitute a system described by a second-order matrix differential equation of the form [9,10,12]

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are of equal size and are called the mass, damping and stiffness matrices, or alternatively the system matrices, $\boldsymbol{f}(t)$ is the input force and $\boldsymbol{x}(t)$ is the response vector. If Eq. (1) is transformed to the modal domain and the structure considered is lightly damped or undamped, $\mathbf{C}=0$, the corresponding eigenvalue equation for the jth mode becomes

$$[-(\omega_j^m)^2\mathbf{M}+\mathbf{K}]\phi_j^m=\mathbf{0} \tag{2}$$

where ω_j^m and ϕ_j^m are the jth measured system natural frequency and its corresponding mode shapes, together known as modal properties. If the natural frequency and mode shape are replaced by analytical quantities then the eigenvalue equation is not exactly satisfied, but

$$[-(\omega_j^a)^2\mathbf{M}+\mathbf{K}]\phi_j^a=\varepsilon_j \tag{3}$$

where ε_j is the residual vector for the jth analytical frequency, ω_j^a, and the corresponding mode shape ϕ_j^a. In this setting, given a set of measured system modal data, D, the finite element model problem is to determine the initial model that realistically approximates the mass and stiffness matrices. Thus the predicted modal data will be as close to the measured modal data as possible (so-called data-match). If these predictions are not sufficiently accurate, then the residual vector will be non-zero and some of the model parameters will need to be modified, giving rise to the FEMUP.

In the next section, we introduce the proposed Bayesian inference approach in the context of finite element model updating.

3. Bayesian inference

Bayesian inference allows one to quantify uncertainties in quantities of interest in a formal way. Bayesian inference is often implemented in two settings; parameter estimation and model selection [4,19]. Parameter estimation is concerned with the plausibility of a given model's parameters based on some observed measurements and this is often carried out using standard Bayes theorem and/or sampling methods e.g. Markov Chain Monte Carlo (MCMC) [7,19,20,27]. In contrast, model selection deals with the mathematical hypothesis of the ability of the model(s) to approximate a particular observed/measured quantity.