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Fast local image inpainting based on the Allen–Cahn model

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ABSTRACT

In this paper, we propose a fast local image inpainting algorithm based on the Allen–Cahn model. The proposed algorithm is applied only on the inpainting domain and has two features. The first feature is that the pixel values in the inpainting domain are obtained by curvature-driven diffusions and utilizing the image information from the outside of the inpainting region. The second feature is that the pixel values outside of the inpainting region are the same as those in the original input image since we do not compute the outside of the inpainting region. Thus the proposed method is computationally efficient. We split the governing equation into one linear equation and one nonlinear equation by using an operator splitting technique. The linear equation is discretized by using a fully implicit scheme and the nonlinear equation is solved analytically. We prove the unconditional stability of the proposed scheme. To demonstrate the robustness and accuracy of the proposed method, various numerical results on real and synthetic images are presented.

1. Introduction

Image inpainting is the process of filling in missing or damaged parts of images based on information from surrounding areas [1]. Up to now, a large number of algorithms have been proposed to solve the image inpainting problem. For the recent survey of the theoretical foundations, the different categories of methods, and illustrations of the main applications about the image inpainting, see the review paper [2] and references therein. Qin et al. [3] proposed an efficient image inpainting approach, which progressively propagates neighboring information into damaged region and can restore sharp edge successfully. In [4], the authors proposed a compact and fast PDE-based inpainting method using anisotropic heat transfer model, which can propagate both the structure and texture information from surrounding region into damaged region simultaneously. Liu and Caselles [5] presented a novel formulation of exemplar-based inpainting [6–8] as a global energy optimization problem, written in terms of the offset map. They also proposed a multiscale graph cuts algorithm to efficiently solve the energy minimization problem. Recently, Ramamurthy et al. [9] regularized the sparse models with manifold projection for image inpainting and Turek et al. [10] used the signal generation model for image inpainting.

Many inpainting algorithms are based on partial differential equations, in which the missing region is filled by diffusing the image information from the known region into the missing region at the pixel level [11–13]. Among them, one of widely applied methods is proposed by Chan and Shen [12]. They proposed a variational framework based on total variation to recover the missing information, which minimizes the following energy functional:

$$\mathcal{E}_{TV}(c)=\int_{\Omega}|\nabla c|d\mathbf{x}+\int_{\Omega}\frac{\lambda(\mathbf{x})}{2}\big(f(\mathbf{x})-c\big)^2d\mathbf{x}, \tag{1}$$

where $\mathbf{x}=(x,y)$, $f(\mathbf{x})$ is a given image, and c is the gray scale image in a domain $\Omega\subset\mathbb{R}^2$. $\Omega_D\subset\Omega$ is the inpainting domain, $\partial\Omega_D$ is the boundary of inpainting domain, and $\Omega\backslash\Omega_D$ is the complement of Ω_D in Ω (see Fig. 1). The fidelity term $\lambda(\mathbf{x})(f(\mathbf{x})-c)^2$ was used to keep the solutions close to the given image in $\Omega\backslash\Omega_D$. For this purpose, $\lambda(\mathbf{x})=0$ if $\mathbf{x}\in\Omega_D$; otherwise $\lambda(\mathbf{x})=\lambda_0$.

The steepest descent equation for the energy functional (1) is given by

$$\frac{\partial c}{\partial t}=\nabla\cdot\left(\frac{\nabla c}{|\nabla c|}\right)+\lambda(\mathbf{x})\big(f(\mathbf{x})-c\big). \tag{2}$$

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1051-2004/

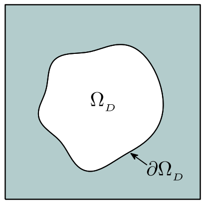

Fig. 1. Schematic illustration of the inpainting domain.

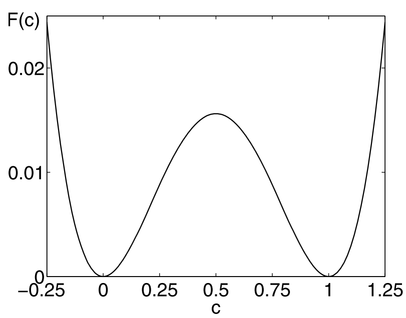

Fig. 2. A double well-potential, $F(c) = 0.25c^2(1-c)^2$.

Hence,

$$\frac{\partial c}{\partial t} = \begin{cases} \nabla \cdot (\frac{\nabla c}{|\nabla c|}) & \text{if } \mathbf{x} \in \Omega_D, \\ \nabla \cdot (\frac{\nabla c}{|\nabla c|}) + \lambda_0(f(\mathbf{x}) - c) & \text{if } \mathbf{x} \in \Omega \backslash \Omega_D. \end{cases} \tag{3}$$

As can be observed, the pixel values in the solution are the same as the values in $\Omega \backslash \Omega_D$, i.e., $c \approx f(\mathbf{x})$. And in the inpainting domain, the pixel values are obtained by curvature-driven diffusions. In [14], Chan and Shen proposed a new inpainting model based on the curvature-driven diffusions to realize the connectivity principle.

$$\frac{\partial c}{\partial t} = \nabla \cdot \left(G\big(\mathbf{x}, |\kappa|\big) \frac{\nabla c}{|\nabla c|} \right) + \lambda\big(f(\mathbf{x}) - c\big). \tag{4}$$

Here,

$$G\big(\mathbf{x}, |\kappa|\big) = \begin{cases} |\kappa|^p & \text{if } \mathbf{x} \in \Omega_D, \\ 1 & \text{if } \mathbf{x} \in \Omega \backslash \Omega_D, \end{cases} \tag{5}$$

where $p \geq 1$ and κ is the curvature, which is given by $\nabla \cdot (\nabla c / |\nabla c|)$. In [13], Esedoḡlu and Shen introduced the Mumford–Shah–Euler model, which is based on the Mumford–Shah image segmentation model [15], to solve the image inpainting problems. Ballester et al. [16] adapted a joint interpolation of vector fields and gray-levels to incorporate the principle of continuity in a variational framework. Li et al. [17] proposed a fast scheme to solve the Chan and Shen's inpainting model [12] with an operator splitting method. Another well-known method was introduced by Bertozzi et al. [18,19], where they proposed an inpainting model which is the modified Cahn–Hilliard (CH) equation

$$\begin{aligned} \mathcal{E}_B(c) &= \mathcal{E}_{B1}(c) + \mathcal{E}_{B2}(c) \\ &= \int_\Omega \left(\frac{F(c)}{\epsilon^2} + \frac{|\nabla c|^2}{2} \right) d\mathbf{x} + \int_\Omega \frac{\lambda}{2} \big(f(\mathbf{x}) - c\big)^2 d\mathbf{x}, \end{aligned} \tag{6}$$

where $F(c) = 0.25c^2(1-c)^2$ is the Helmholtz free energy density (see Fig. 2). The term $F(c)$ is a force that makes c to be approximately 0 or 1. $|\nabla c|^2$ is a gradient energy, ϵ is the gradient energy coefficient related to the interfacial energy.

By a superposition of gradient descent with respect to H^{-1} inner product for the energy $\mathcal{E}_{B1}$ and gradient descent with respect to L^2 inner product for the energy $\mathcal{E}_{B2}$, the authors in [18,19] proposed the following model:

$$\frac{\partial c}{\partial t} = \Delta \left(\frac{F'(c)}{\epsilon^2} - \Delta c \right) + \lambda\big(f(\mathbf{x}) - c\big). \tag{7}$$

We note that if $\lambda = 0$ in Eq. (7), then the equation becomes the classical CH equation [20], which has been used as a mathematical model to investigate the phase separation of binary mixture under quenching below a critical temperature. For physical, mathematical, and numerical derivations of the CH equation, see the recent review paper [21].

In this paper, we propose a new effective and accurate image inpainting method which is based on the local Allen–Cahn equation [22]. The equation has been used in solving problems in image processing [23–27]. By using the Allen–Cahn equation, we can perform fast image inpaintings, because its fast and accurate hybrid numerical solver is available [28]. It should be pointed that our model is the extension of Chan and Shen's model [12], since Allen–Cahn equation describes the motion of mean curvature flow. Compared to Cahn–Hilliard equation [18,19], we choose to use the Allen–Cahn equation, since its numerical treatment is simpler than that of the Cahn–Hilliard type, which involves fourth-order differential operators. The outline of this paper is the following. In Section 2, the governing equations for the image inpainting are presented. In Section 3, we describe the proposed operator splitting algorithm and give a detailed proof for its unconditional stability. In Section 4, we present computational examples to demonstrate the efficiency and robustness of our proposed method. Finally, conclusions are drawn in Section 5.

2. Description of the proposed model

The inpainting algorithms can be summarized as: First, the missing region is filled by diffusing the image information from the known region into the missing region at the pixel level. Second, the image information in the known region should be close to the given image. To reduce the computational cost and keep the accuracy of the inpainting algorithm, we propose the following equations:

$$\frac{\partial c}{\partial t}(\mathbf{x}, t) = \begin{cases} -F'(c(\mathbf{x}, t))/\epsilon^2 + \Delta c(\mathbf{x}, t) & \text{if } \mathbf{x} \in \Omega_D, \\ 0 & \text{otherwise}. \end{cases} \tag{8}$$

$$c(\mathbf{x}, 0) = \begin{cases} 0.5 & \text{if } \mathbf{x} \in \Omega_D, \\ \frac{f(\mathbf{x}) - f_{\min}}{f_{\max} - f_{\min}} & \text{otherwise}. \end{cases} \tag{9}$$

$$\mathbf{n} \cdot \nabla c(\mathbf{x}, t) = 0, \quad \mathbf{x} \in \partial\Omega, \tag{10}$$

where $f_{\max}$ and $f_{\min}$ are the maximum and the minimum values of the given image, respectively, and $\mathbf{n}$ is the unit normal vector to $\partial\Omega$. Thus, we have the normalized image data $c(\mathbf{x}, 0) \in [0, 1]$. Eq. (8) is also called as the Allen–Cahn equation [22], which is widely applied in image processing due to the motion by mean curvature. We choose the AC equation because it has intrinsic smoothing effect on interfacial transition layers and its fast and accurate hybrid numerical solver is available [28].

For the initial condition, we can choose different initial guesses for the inpainting domain Ω_D because the solution of missing regions will be defined by the information of the known region at the equilibrium solution. In this work, to reduce the numerical iterations, we set $c(\mathbf{x}, 0) = 0.5$ in the inpainting domain.

3. Numerical solution algorithm

3.1. Proposed operator splitting algorithm

In this section, we propose an operator splitting method to get an unconditionally stable numerical method for solving the proposed algorithm. Let the computational domain Ω be $[1, N_x] \times$