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Recursive least squares parameter identification algorithms for systems with colored noise using the filtering technique and the auxilary model ☆

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ABSTRACT

This paper focuses on the parameter estimation problems of output error autoregressive systems and output error autoregressive moving average systems (i.e., the Box–Jenkins systems). Two recursive least squares parameter estimation algorithms are proposed by using the data filtering technique and the auxiliary model identification idea. The key is to use a linear filter to filter the input–output data. The proposed algorithms can identify the parameters of the system models and the noise models interactively and can generate more accurate parameter estimates than the auxiliary model based recursive least squares algorithms. Two examples are given to test the proposed algorithms.

1. Introduction

The development of information and communication technology has had a tremendous impact on our lives, e.g., the information filtering, optimization and estimation techniques [1–4]. In the areas of signal processing and system identification, the observed output signals always contain disturbances from process environments [5–8]. The disturbances are of different forms (white noise or colored noise). It is well known that the conventional recursive least squares (RLS) method generates biased parameter estimates due to correlated noise or colored noise [9]. Thus the identification of output error models with colored noise has attracted many research interests [10]. The bias correction methods have been considered very effective to deal with the output error models with colored noise [11,12]. However, the bias correction methods ignore the estimation of the noise models [13]. In this paper, we propose new identification methods for estimating the parameters of the system model and the noise model.

Since the noise in real life can be fitted by the autoregressive (AR) models, the moving average (MA) models [14,15] or the autoregressive moving average (ARMA) models [16], this paper considers the output error (OE) model with AR noise as shown in Fig. 1 (the OEAR model for short), which can be expressed as

$$y(t)=\frac{B(z)}{A(z)}u(t)+\frac{1}{C(z)}v(t), \tag{1}$$

where $\{u(t)\}$ and $\{y(t)\}$ are the system input and output sequences, respectively, $\{v(t)\}$ is a white noise sequence with zero mean and variance σ^2, and $A(z)$, $B(z)$ and $C(z)$ are polynomials in the unit backward shift operator z^{-1} $[z^{-1}y(t)=y(t-1)]$:

$$A(z):=1+a_1z^{-1}+a_2z^{-2}+\ldots+a_{n_a}z^{-n_a},$$
$$B(z):=b_1z^{-1}+b_2z^{-2}+\ldots+b_{n_b}z^{-n_b},$$
$$C(z):=1+c_1z^{-1}+c_2z^{-2}+\ldots+c_{n_c}z^{-n_c}.$$

Assume that the orders n_a, n_b and n_c are known, and $u(t)=0$, $y(t)=0$ and $v(t)=0$ for $t\leqslant 0$. The coefficients a_i, b_i and c_i are the parameters to be estimated from the input–output data $\{u(t),y(t)\}$.

The model in (1) can be transformed into a new controlled autoregressive moving average (CARMA) form,

$$A(z)C(z)y(t)=B(z)C(z)u(t)+A(z)v(t),$$

or

$$A'(z)y(t)=B'(z)u(t)+D(z)v(t),\qquad A'(z):=A(z)C(z),$$
$$B'(z):=B(z)C(z),\qquad D(z):=A(z).$$

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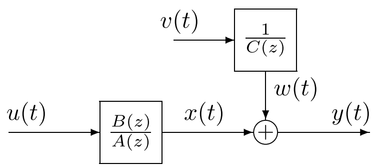

Fig. 1. The output error autoregressive system.

This CARMA model can be identified using the recursive extended least squares algorithm [9,17]. However, the model in (1) only contains $(n_a+n_b+n_c)$ unknown parameters, while this new model contains $(2n_a+n_b+2n_c)$ parameters, resulting in the increment of the computation load of identification algorithms. Moreover, some extra computation is required to compute the estimates of the parameters b_i and c_i.

In practical industries, there exist many unmeasurable variables in systems, such as the state variables [18] and the inner variables or the noise-free outputs. In general, one can use the outputs of an appropriate auxiliary model to replace the unmeasurable variables for identification. The auxiliary model identification idea can be applied to linear systems containing the unknown variables in the information vectors [19,20], nonlinear systems, dual-rate/multirate systems [21,22], and missing-data systems or systems with scarce measurements [23,24]. Recently, Chen et al. presented a data filtering based least squares iterative algorithm for parameter identification of output error autoregressive systems [25].

On the basis of the work in [25–27], this paper investigates the recursive identification problems of the OEAR models and the Box–Jenkins models using the filtering technique. Two-stage recursive least squares algorithms are proposed through filtering the input–output data. Since the OEAR models and the Box–Jenkins models involve the system models (the OE part) and the noise models (the AR or ARMA part), the proposed algorithms can generate the parameter estimates of the system models and the noise models.

The rest of this paper is organized as follows. Section 2 gives the auxiliary model identification algorithm for OEAR systems. Section 3 analyzes the convergence analysis of the auxiliary model based recursive generalized least squares algorithm. Section 4 derives a parameter estimation algorithm based on the data filtering technique. Section 5 gives simply a filtering based identification algorithm for Box–Jenkins systems. Section 6 provides two examples to show the effectiveness of the proposed algorithms. Finally, some concluding remarks are given in Section 7.

2. The auxiliary model based recursive generalized least squares algorithm

Define the noise-free output $x(t)$ and the noise term $w(t)$ as

$$x(t) := \frac{B(z)}{A(z)}u(t), \qquad w(t) := \frac{1}{C(z)}v(t), \tag{2}$$

and the parameter vector $\boldsymbol{\vartheta}$ and the information vector $\boldsymbol{\phi}(t)$ as

$$\boldsymbol{\vartheta} := \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{c} \end{bmatrix} \in \mathbb{R}^n, \quad n := n_a + n_b + n_c,$$

$$\boldsymbol{\theta} := [a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_a+n_b},$$

$$\boldsymbol{c} := [c_1, c_2, \dots, c_{n_c}]^{\mathrm{T}} \in \mathbb{R}^{n_c},$$

$$\boldsymbol{\phi}(t) := \begin{bmatrix} \boldsymbol{\varphi}(t) \\ \boldsymbol{\psi}(t) \end{bmatrix} \in \mathbb{R}^n,$$

$$\boldsymbol{\varphi}(t) := \big[-x(t-1), -x(t-2), \dots, -x(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)\big]^{\mathrm{T}} \in \mathbb{R}^{n_a+n_b},$$

$$\boldsymbol{\psi}(t) := \big[-w(t-1), -w(t-2), \dots, -w(t-n_c)\big]^{\mathrm{T}} \in \mathbb{R}^{n_c}.$$

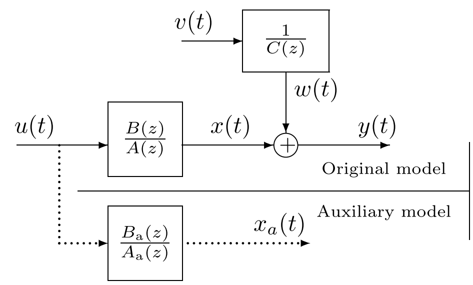

Fig. 2. The output error auto-regressive systems with the auxiliary model.

Eqs. (2) and (1) can be written as

$$\begin{aligned} x(t) &= \big[1 - A(z)\big]x(t) + B(z)u(t) \\ &= \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}, \end{aligned} \tag{3}$$

$$\begin{aligned} w(t) &= \big[1 - C(z)\big]w(t) + v(t) \\ &= \boldsymbol{\psi}^{\mathrm{T}}(t)\boldsymbol{c} + v(t), \end{aligned} \tag{4}$$

$$\begin{aligned} y(t) &= x(t) + w(t) \\ &= \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\vartheta} + v(t). \end{aligned} \tag{5}$$

A difficulty of identification is that $\boldsymbol{\phi}(t)$ contains the unknown inner term $x(t-i)$ and the unmeasurable noise term $w(t-i)$. An effective method of estimating the parameter vector $\boldsymbol{\vartheta}$ is to employ the auxiliary model identification idea in [19,28] as shown in Fig. 2, where $x_{\mathrm{a}}(t) := \frac{B_{\mathrm{a}}(z)}{A_{\mathrm{a}}(z)}u(t)$ is the output of the auxiliary model. The unknown term $x(t-i)$ is replaced with the output $x_{\mathrm{a}}(t-i)$ of the auxiliary model and the unknown noise term $w(t-i)$ is replaced with its estimate $\hat{w}(t-i)$ for parameter estimation. Define

$$\hat{\boldsymbol{\phi}}(t) := \begin{bmatrix} \boldsymbol{\varphi}_{\mathrm{a}}(t) \\ \hat{\boldsymbol{\psi}}(t) \end{bmatrix} \in \mathbb{R}^n,$$

$$\boldsymbol{\varphi}_{\mathrm{a}}(t) := \big[-x_{\mathrm{a}}(t-1), -x_{\mathrm{a}}(t-2), \dots, -x_{\mathrm{a}}(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)\big]^{\mathrm{T}} \in \mathbb{R}^{n_a+n_b},$$

$$\hat{\boldsymbol{\psi}}(t) := \big[-\hat{w}(t-1), -\hat{w}(t-2), \dots, -\hat{w}(t-n_c)\big]^{\mathrm{T}} \in \mathbb{R}^{n_c}.$$

Referring to the methods in [19,28], we can obtain the auxiliary model based recursive generalized least squares (AM-RGLS) algorithm for generating the estimate $\hat{\boldsymbol{\vartheta}}(t)$ of $\boldsymbol{\vartheta}$:

$$\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}(t)\big[y(t) - \hat{\boldsymbol{\phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1)\big], \tag{6}$$

$$\boldsymbol{L}(t) = \boldsymbol{P}(t)\hat{\boldsymbol{\phi}}(t) = \boldsymbol{P}(t-1)\hat{\boldsymbol{\phi}}(t)\big[1 + \hat{\boldsymbol{\phi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\phi}}(t)\big]^{-1}, \tag{7}$$

$$\begin{aligned} \boldsymbol{P}(t) &= \boldsymbol{P}(t-1) - \boldsymbol{L}(t)\boldsymbol{L}^{\mathrm{T}}(t)\big[1 + \hat{\boldsymbol{\phi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\phi}}(t)\big], \\ \boldsymbol{P}(0) &= p_0\boldsymbol{I}, \end{aligned} \tag{8}$$

$$\hat{\boldsymbol{\phi}}(t) = \begin{bmatrix} \boldsymbol{\varphi}_{\mathrm{a}}(t) \\ \hat{\boldsymbol{\psi}}(t) \end{bmatrix}, \tag{9}$$

$$\boldsymbol{\varphi}_{\mathrm{a}}(t) = \big[-x_{\mathrm{a}}(t-1), -x_{\mathrm{a}}(t-2), \dots, -x_{\mathrm{a}}(t-n_a), u(t-1), u(t-2), \dots, u(t-n_b)\big]^{\mathrm{T}}, \tag{10}$$

$$\hat{\boldsymbol{\psi}}(t) = \big[-\hat{w}(t-1), -\hat{w}(t-2), \dots, -\hat{w}(t-n_c)\big]^{\mathrm{T}}, \tag{11}$$

$$x_{\mathrm{a}}(t) = \boldsymbol{\varphi}_{\mathrm{a}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t), \tag{12}$$

$$\hat{w}(t) = y(t) - x_{\mathrm{a}}(t), \tag{13}$$

$$\hat{\boldsymbol{\vartheta}}(t) = \big[\hat{\boldsymbol{\theta}}^{\mathrm{T}}(t), \hat{c}_1(t), \hat{c}_2(t), \dots, \hat{c}_{n_c}(t)\big]^{\mathrm{T}}, \tag{14}$$

$$\hat{\boldsymbol{\theta}}(t) = \big[\hat{a}_1(t), \hat{a}_2(t), \dots, \hat{a}_{n_a}(t), \hat{b}_1(t), \hat{b}_2(t), \dots, \hat{b}_{n_b}(t)\big]^{\mathrm{T}}. \tag{15}$$