Digital Signal Processing 26 (2014) 50–70

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Component statistical analysis of second order hidden periodicities

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ARTICLE INFO

Article history:
Available online 19 December 2013

Keywords:
Gaussian periodically correlated random processes
Component method
Hidden periodicity
Frequency estimation
Small parameter
Vibration

ABSTRACT

The component method is applied to define estimators of the periods for Gaussian periodically correlated random processes (mathematical model of stochastic oscillations). The properties of these period estimators are obtained using some small parameter method and the rate of convergence is shown to be optimal. Specific results for the simplest models of periodically correlated process are presented. Finally the method is illustrated with a simulated sequence and a real life vibration signal.

1. Introduction

In signal processing, the analysis of the first moment behavior is not always sufficient to detect periodicities hidden in the structure of the signal [7]. When the signal has a constant mean, we should use the second order statistics. The second order periodic nonstationarity of the random component is caused by stochastic modulation of harmonic functions [1,2]. This modulation often leads to wide band processes and cannot be characterized with a spectral density function. Moreover the presence or absence of periodicity properties in a signal cannot be determined only by studying the behavior of the covariance function by time lag as proposed in [3–5]. The stationary model is useless to determine the periodicity in time of the covariance function since within the stationary approach the time variability of the covariance is ignored. The methods of hidden periodicity detection should test the time changes of statistical properties. This main innovation is proposed in [6,7] and developed in [8–10]. The statistics that characterize periodic nonstationarity of a signal are set as principles of searching for hidden periodicities [11–13].

The simplest model of hidden periodicity, the *periodically correlated random real-valued process* (PC process), also called in the Signal Processing community as cyclostationary signal, $\xi = \{\xi(t),\ t \in \mathbb{R}\}$ has by definition [14–16], a periodic mean function $m(t) = E[\xi(t)] = m(t+\theta)$ and a periodic covariance $b(t,u) = E[\bar{\xi}(t)\bar{\xi}(t+u)] = b(t+\theta,u)$ with the same period $\theta > 0$. Here $\bar{\xi}(t) = \xi(t) - m(t)$. If

$$\int_0^\theta \left|m(t)\right| dt < \infty \quad \text{and} \quad \int_0^\theta \left|b(t,u)\right| dt < \infty,$$

then we may use the following Fourier series representations:

$$m(t) = \sum_{k\in\mathbb{Z}} m_k e^{ik\omega_\theta t} = m_0 + \sum_{k\in\mathbb{N}} \left(m_k^c \cos(k\omega_\theta t) + m_k^s \sin(k\omega_\theta t)\right),$$

$$b(t,u) = \sum_{k\in\mathbb{Z}} B_k(u) e^{ik\omega_\theta t}$$
$$= B_0(u) + \sum_{k\in\mathbb{N}} \left(B_k^c(u)\cos(k\omega_\theta t) + B_k^s(u)\sin(k\omega_\theta t)\right)$$

where $\omega_\theta = 2\pi/\theta$, the terms $B_k(u)$ are known as the covariance components [1,2]. For the estimation of the characteristics, different methods can be used: the coherent method [15–17], the component method [18], the least squares method [19] and linear filtration method [2,20]. See also [6,7].

For the estimation of the period θ of the mean function of a PC process, the cosine and sine Fourier transforms

$$\widehat{m}_l^c(\tau) = \frac{1}{T}\int_{-T}^{T} \xi(t)\cos(l\omega_\tau t)\,dt,$$

$$\widehat{m}_l^s(\tau) = \frac{1}{T}\int_{-T}^{T} \xi(t)\sin(l\omega_\tau t)\,dt,$$

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1051-2004/$ – see front matter
http://dx.doi.org/10.1016/j.dsp.2013.12.002

where $\omega_\tau = 2\pi/\tau$, τ is a test period and $l > 0$, have been analyzed as functions of τ. When $\tau = \theta$ and $T = N\theta$, the statistics $\widehat{m}_l^c(\tau)$ and $\widehat{m}_l^s(\tau)$ are unbiased estimators of respectively m_l^c and m_l^s. Moreover, under the condition

$$\lim_{|u|\to\infty} b(t,u) = 0, \tag{1}$$

they are consistent [17,18]. The period estimators considered in [7] are the arguments of the functionals for which the extreme values are attained. Thus they are obtained as solutions of the nonlinear stochastic equations

$$\frac{d\widehat{m}^c(\tau)}{d\tau} = 0, \quad \text{respectively} \quad \frac{d\widehat{m}^s(\tau)}{d\tau} = 0.$$

The solutions are approximated by polynomial functions with a small parameter that tends to zero as the realization length $2T$ goes to the infinity. Under the condition (1) these estimators are asymptotically unbiased and consistent. Here we will extend this idea to the second order moment characterizations in order to identify the stochastic recurrence of the processes. The method proposed in [7] for the study of the solutions of nonlinear equation is called *small parameter method* [22,23]. Notice that this method combined with the likelihood method was successfully applied in the parameter estimation problem for signals with additive noise [23].

To estimate the period of a PC process with the maximum likelihood method [24] the problem could be parameterized using the harmonic representation [15,16]

$$\xi(t) = \sum_{k\in\mathbb{Z}} \xi_k(t) e^{ik\omega_\theta t}, \tag{2}$$

where $\xi_k = \{\xi_k(t),\ t \in \mathbb{R}\}$, $k \in \mathbb{Z}$, is a family of jointly stationary complex valued processes. The period estimate in this case is obtained by solving the likelihood equation and can be analyzed with the small parameter method. Then taking into account the prior knowledge about the probability structure of the PC process under study, it can be shown that the most probable estimate of the period is asymptotically unbiased and consistent. Nevertheless, at the early stage of investigation the information on the probability structure of the process is missing, that is the reason why we prefer methods which are less efficient but which can be applied to a larger class of processes [6,7]. Here we consider methods based on the coherent and component estimators for PC process [16,17,20]. There is a large amount of work dedicated to the problem of estimation of the periods (frequencies) for a polyharmonic oscillation noised by a stationary random process (additive model) $\xi(t) = f(t) + \eta(t)$, where $f(t)$ is an almost periodic function with a finite number of harmonics [3–5,25,26]. In the case of white noise the solution of the problem is trivial. Several significant solutions are obtained in the case of colored noise under special conditions. For instance in P. Stoica's works [25,26] some simple formulae for the asymptotic Cramér–Rao bound are obtained and it is shown that the variance of the estimators is as small as T^{-3}, where $2T$ is the length of the observation of the process. These results are based on the stationary approach thus cannot be applied to PC process.

Synopsis: The paper is organized as follows. In Section 2 we introduce and analyze the period estimator as an extreme point of the cosine and sine covariance transform or/and the combination of these transformations. The period estimates are investigated as the solutions of nonlinear equations using the small parameter method. The convergence in mean square is proved, and approximations of the bias and the variances of the estimators are obtained. Section 2 is devoted to the analysis of the covariance component estimators, and in Section 3 we consider the correlation estimators. In each section, the proposed methods are illustrated with one of the simplest signal models: the multiplicative model. The final section concerns the application of the developed method to simulated data from a quadrature model, and to real-life vibroacoustic data.

2. Covariance functionals

2.1. Cosine and sine covariance functionals

In the following we assume that the mean of the signal is constant, so does not contain any periodic component, thus $E[\xi(t)] = m$ for all t and for some known $m \in \mathbb{R}$. Then we are going to consider the cosine and sine covariance components $B_l^{c,s}(u)$ and for estimating the period we introduce the statistics

$$\widehat{B}_l^c(u,\tau) = \frac{1}{T}\int_{-T}^{T} \bar{\xi}(t)\bar{\xi}(t+u)\cos(l\omega_\tau t)\,dt, \tag{3}$$

$$\widehat{B}_l^s(u,\tau) = \frac{1}{T}\int_{-T}^{T} \bar{\xi}(t)\bar{\xi}(t+u)\sin(l\omega_\tau t)\,dt, \tag{4}$$

and $\widehat{B}_0(u,\tau) = \widehat{B}_0^c(u,\tau)/2$, where $\bar{\xi}(t) = \xi(t) - m$ and $l \geqslant 0$. The deterministic components of these transforms are equal to

$$S_l^c(u,\tau) = E\big[\widehat{B}_l^c(u,\tau)\big] = \frac{1}{T}\int_{-T}^{T} b(t,u)\cos(l\omega_\tau t)\,dt, \tag{5}$$

$$S_l^s(u,\tau) = E\big[\widehat{B}_l^s(u,\tau)\big] = \frac{1}{T}\int_{-T}^{T} b(t,u)\sin(l\omega_\tau t)\,dt. \tag{6}$$

We know that

$$\lim_{T\to\infty} S_l^{c,s}(u,\tau) = B_l^{c,s}(u,\tau) = \begin{cases} B_k^{c,s}(u) & \text{if } \tau = l\theta/k \text{ for some } k > 0, \\ 0 & \text{otherwise} \end{cases}$$

and

$$\limsup_{T\to\infty} T\big|S_l^{c,s}(u,\tau) - B_l^{c,s}(u,\tau)\big| < \infty.$$

If

$$\int_0^\theta \big|b(t,u)\big|\,dt < \infty,$$

then

$$b(t,u) = B_0(u) + \sum_{k=1}^{\infty}\big\{B_k^c(u)\cos(k\omega_\theta t) + B_k^s(u)\sin(k\omega_\theta t)\big\}, \tag{7}$$

the series converging in $L^1[0,\theta]$ with respect to t, for each u. Thus, taking into account the relations (5) and (6) we obtain

$$S_l^c(u,\tau) = 2B_0(u)J_0(l\omega_\tau T) + \sum_{k=1}^{\infty} B_k^c(u)\big\{J_0\big((k\omega_\theta + l\omega_\tau)T\big) + J_0\big((k\omega_\theta - l\omega_\tau)T\big)\big\},$$

$$S_l^s(u,\tau) = \sum_{k=1}^{\infty} B_k^s(u)\big\{J_0\big((k\omega_\theta - l\omega_\tau)T\big) - J_0\big((k\omega_\theta + l\omega_\tau)T\big)\big\},$$