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Model updating of lattice structures: A substructure energy approach

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ABSTRACT

Model updating is an inverse problem to identify and correct uncertain modeling parameters, which leads to better predictions of the dynamic behavior of a target structure. This study presents a direct physical property adjustment method and the substructure energy approach, which can accurately update the stiffness and mass matrices of lattice structures using incomplete eigenvectors measured from critical substructures. For validation, the proposed method is applied to update models of a mass-spring system, a two-dimensional, and a three-dimensional lattice structure.

1. Introduction

With scalable geometry and repetition characteristics, lattice structures are widely employed in aerospace, mechanical, marine, and civil systems [1]. Their structural properties depend on the actual size of the unit cell, enabling applications in both the small- and large-scale structural systems [2]. For the purpose of evaluating the mechanical integrity of a lattice structure, numerical (mathematical) model needs to be established via the use of the finite element (FE) method. In order to improve the correlation between the FE model and experimentally measured data, model updating is emerging as one of the most important topics in the area of modal testing [3], and the deduced optimal structural models can be used for structural health monitoring applications [4,5]. In essence, model updating identifies and corrects uncertain modeling parameters, and integrates the information into a FE model that could better evaluate the dynamic behaviors of a target structure. There are two broad approaches for updating the system matrices based on the type of parameters that need updating as well as the available data measured from experiment: (a) updating from modal data and (b) updating directly from the force-response measurements. This study focuses on the first category.

The conventional modal-based updating approaches usually rely on either the matrix adjustment method or the physical property adjustment method [6,7]. The first method computes directly the changes to the mass and stiffness matrices, and the resulting models become "abstract representations" and cannot be interpreted in a physical way [8]. The second method seeks physical quantities for each individual element or associated parameter; this method is closely related to the model-based methods for damage determination, which can be used to quantify the location and extent of damage [9].

With respect to the physical property adjustment method, there are two widely used approaches: (a) iterative methods and (b) non-iterative methods. For iterative methods, spatially complete measured modes are not necessary, but there are

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several disadvantages: a very expensive computation effort is required, the modal information must be obtained from the same order mode of both original and real models, and analytical and measured modes need to be consistent in scale [10]. To avoid these problems, Hu et al. [11] developed a non-iterative physical property adjustment method that can update the stiffness and mass matrices simultaneously. This method, however, requires the spatial complete modal data to be measured from the updated structure. When this method is applied to large-scale structures such as lattice structures, it is not practical to identify all of the unknown parameters in the structure at the same time since ill-conditioning and non-uniqueness in the solution of inverse problems appear as inevitable difficulties, and it is not practical to measure all degrees of freedom of the structural modes [12][1] since the measurement may be limited to a small number of locations of the actual structure.

An alternative process to perform a non-iterative physical property adjustment approach is by applying one algorithm to identify the critical subsystems with uncertain modeling parameters (first step), and then implementing another algorithm to estimate the uncertain modeling parameters after the critical subsystems have been identified (second step). Several effective substructure identification methods [12–14], which are well established in the literature, may be used for the first step of identifying the critical parts (subsystems) with uncertain physical parameters. The complete structure is divided into several substructures and the analysis is concentrated on a critical subsystem with a smaller number of elements or associated uncertain parameters. After the critical parts are identified, one may utilize the modal information of these parts to estimate the correction coefficients of the uncertain modeling parameters. However, for such a second step (which is the focus of the current research), the existing non-iterative physical property adjustment methods (e.g. [11,15]) still require the knowledge of the complete modal information of the entire structure, which is not only impractical but also makes the previous substructure identification process ineffective. To the best of the authors' knowledge, a non-iterative physical property adjustment method is not yet available, which could directly utilize the spatially incomplete modal information to update modeling parameters.

In this paper, the main objective is to develop a non-iterative physical property adjustment method, the substructure energy approach (SEAp), which can accurately estimate the stiffness and mass matrix correction factors for lattice structures using spatially incomplete measurements (after the critical subsystems have been identified using established methods, e.g. [12–14]). The model updating method involves a set of linear simultaneous equations that are deduced from the energy functional of substructure models and substructure modes; the correction factors can be solved without iteration. The accuracy and effectiveness of the proposed SEAp are validated using three numerical examples, a mass-spring system that has been studied recently by many researchers using various updating approaches [8,11,15,16], a two-dimensional (2D) lattice structure and a three-dimensional (3D) lattice structure where for each element the mass and stiffness correction factors are determined from SEAp based on the incomplete measured modal information.

2. Substructure energy approach (SEAp)

The structure of the beam-type lattice structure is composed of a sequence of identical unit cells repeating along the axial direction (and also across the thickness if necessary). Each cell is composed of beam elements. In the following text, the term "analytical model" refers to the original, unmodified model, and the term "true model" refers to the real structure whose information is measured at critical locations, which provides the basis for correcting the analytical model. The schematic of the lattice system with identical unit cells is shown in Fig. 1. The total energy functional of the analytical model under consideration may be expressed as

$$\Psi = \sum_{j=1}^{N} \psi_j \tag{1}$$

where ψ_j is the energy functional of the jth substructure and N the total number of substructures. In general, the substructure energy functional may be defined as

$$\psi_j = \frac{1}{2}\boldsymbol{\delta}_j^T \mathbf{K}_j \boldsymbol{\delta}_j - \frac{1}{2}\omega^2 \boldsymbol{\delta}_j^T \mathbf{M}_j \boldsymbol{\delta}_j - \boldsymbol{\delta}_j^T \mathbf{F}_j \tag{2}$$

where $\omega^2 \mathbf{M}_j \boldsymbol{\delta}_j$ is considered as the inertia force in addition to the loading factor. $\mathbf{K}_j$ and $\mathbf{M}_j$ denote the stiffness and the mass matrices of the jth substructure, $\boldsymbol{\delta}_j$ and $\mathbf{F}_j$ denote the displacement and loading vectors for the jth substructure, respectively, and ω is the circular frequency of vibration.

For the analytical lattice structure, the stiffness and mass matrix of the substructure is repetitive, i.e. $\mathbf{K}_j = \mathbf{K}_{sub}$ and $\mathbf{M}_j = \mathbf{M}_{sub}$. Let $\mathbf{K}_j^e$ and $\mathbf{M}_j^e$ be the stiffness and mass matrices of the jth substructure of the true (experimental) model,

[1] If a structural model is available, the mode may be expanded to obtain the spatially complete version; however, expanding mode shape may bring in additional error into the mode information [2].