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Operational modal analysis by updating autoregressive model

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ABSTRACT

This paper presents improvements of a multivariable autoregressive (AR) model for applications in operational modal analysis considering simultaneously the temporal response data of multi-channel measurements. The parameters are estimated by using the least squares method via the implementation of the QR factorization. A new noise rate-based factor called the Noise rate Order Factor (NOF) is introduced for use in the effective selection of model order and noise rate estimation. For the selection of structural modes, an orderwise criterion called the Order Modal Assurance Criterion (OMAC) is used, based on the correlation of mode shapes computed from two successive orders. Specifically, the algorithm is updated with respect to model order from a small value to produce a cost-effective computation. Furthermore, the confidence intervals of each natural frequency, damping ratio and mode shapes are also computed and evaluated with respect to model order and noise rate. This method is thus very effective for identifying the modal parameters in case of ambient vibrations dealing with modern output-only modal analysis. Simulations and discussions on a steel plate structure are presented, and the experimental results show good agreement with the finite element analysis.

1. Introduction

Modal analysis identification techniques give useful information on modal parameters to understand the dynamic behavior of a structure [1]. However, in many cases which present nonlinear behaviors, the modal parameters have to be estimated under operating conditions and very often the excitations cannot be measured [2]. We must thus consider the operational modal analysis. Although the identification technique can be conducted both in the frequency [3] or time [4–6] domains, it is seen that the time domain is more suitable for operational modal analysis and can be classified into two groups. The first group lies in the fitting of response correlation functions, including the Ibrahim Time Domain (ITD) method [7], the Least Squares Complex Exponential (LSCE) method [8], the Covariance-driven Stochastic Subspace Identification (SSI-COV) method [9], and several modified versions of these methods for more suitable applications, particularly under harmonic excitations [10–13]. Other methods, based on parametric models, involve choosing a mathematical model to idealize the structural dynamic responses, including AutoRegressive Moving Average (ARMA) and AutoRegressive (AR) models [14–18]. For these autoregressive methods, a system identification algorithm is needed for estimating the model parameters. Among them, the Prediction Error Method (PEM) [19] is a common technique based on either the least squares estimate or on the Gauss–Newton iterative search. When the mode identification is necessary, multiples sensors have to be simultaneously recorded, and several applications of the multivariate AR model can be found using the ordinary least squares in the form of normal equations [20–22] or the Levinson algorithm [23]. The PEM iterative

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Nomenclature

$\mathbf{A}_i$	matrix of parameters relating the output $\mathbf{y}(t-i)$ to $\mathbf{y}(t)$
d	vector dimension or number of sensors
$\hat{\mathbf{D}}$	estimated covariance matrix of the deterministic part
$\mathbf{e}(t)$	the residual vector of all output channels
$\hat{\mathbf{E}}$	estimated covariance matrix of the error part
f_i	natural frequency
$\mathbf{I}$	unity matrix
$\mathbf{K}$	data matrix
$\mathbf{K}_{1,2}$	subdivided data matrices
$\mathbf{K}^*$	added data columns matrix
$\mathbf{L}$	complex eigenvectors matrix
N	number of available data samples
p	model order
$p_{\text{eff(com)}}$	efficient (computing) model order
$\mathbf{Q}$	orthogonal factor matrix of the QR factorization
$\mathbf{Q}_a$	Q factor of the added columns factorization
$\mathbf{R}$	upper-diagonal factor matrix of the QR factorization
$\mathbf{R}_a$	R factor of the added columns factorization
$\mathbf{R}_{ij}$	submatrices of $\mathbf{R}$
$\mathbf{R}_{22}^{*(**)}$	factorized matrices of $\mathbf{R}_{22}$ factor of the order updating
t	time index
T_s	sampling period
$\mathbf{T}_{1,2,3}$	R factor corresponding to added data
u_i	discrete complex eigenvalue
$\mathbf{y}(t-i)$	the output vector with time delay $i \times T_s$
$\mathbf{z}(t)$	the regressor for the output vector $\mathbf{y}(t)$
ζ_i	damping ratio
$\mathbf{\Theta}$	real mode shapes matrix
λ_i	continuous complex eigenvalue
$\mathbf{\Lambda}$	model parameters matrix
π	Pi number
$\mathbf{\Pi}$	state matrix
$\mathbf{\Psi}$	complex mode shapes matrix
$^{(p)}$	parameter at order p
$\hat{}$	estimated value
$^{\mathrm{T}}$	matrix/vector transpose
$\bar{}$	conjugated transpose
$\lvert\rvert$	absolute value
Im(…)	imaginary part
Re(…)	real part
Trace(…)	trace norm of a matrix

method, generally used to search for minimization, requires intensive computation and initial start-up values which are normally calculated using the least squares method. Furthermore, in some cases, the local minimization problem poses a big challenge [19].

In this paper, the multivariate autoregressive model is expressed in a convenient fashion for computation. The QR factorization gives an easy, fast and well-conditioned formulation for the least squares estimate of model parameters, and can be effectively updated with respect to the model order. A new factor based on the separation and evolution of the signal and noise is developed for the model order selection and noise rate estimation. The modal parameters are derived using the eigen-decomposition of the state matrix. An orderwise version of the correlation criterion called the Order Modal Assurance Criterion (OMAC) is defined for a user-friendly selection of modes. For interest on uncertainty in the parameter estimates, the confidence intervals for each modal parameter are computed. Finally, the method is applied both on simulated and experimental data of a steel plate in comparison to finite element method.

2. Vector-autoregressive model and its order updating

In operational modal analysis, we assume that the excitation is unknown. Since the modal analysis is conducted by using several d channels of measurements synchronized for data acquisition at sampling period T_s, a multivariate autoregressive model of pth order and dimension d can be utilized to fit the measured data [14,19].

$$\mathbf{y}(t) = \mathbf{\Lambda}\mathbf{z}(t) + \mathbf{e}(t) \tag{1}$$

where $\mathbf{\Lambda} = [-\mathbf{A}_1 \quad -\mathbf{A}_2 \quad \cdots \quad -\mathbf{A}_p]$ of size $d \times dp$ is the parameters matrix, $\mathbf{A}_i$ of size $d \times d$ is the matrix of parameters relating the output $\mathbf{y}(t-i)$ to $\mathbf{y}(t)$, $\mathbf{z}(t)$ of size $dp \times 1$ is the regressor for the output vector $\mathbf{y}(t)$, $\mathbf{z}(t)^{\mathrm{T}} = [\mathbf{y}(t-1)^{\mathrm{T}} \quad \mathbf{y}(t-2)^{\mathrm{T}} \quad \cdots \quad \mathbf{y}(t-p)^{\mathrm{T}}]$, $\mathbf{y}(t-i)$ of size $d \times 1$ (i=1:p) is the output vector with delayed time $i \times T_s$, $\mathbf{e}(t)$ of size $d \times 1$ is the residual vector of all output channels considered as the error of model.

If N consecutive output vectors of the responses from $\mathbf{y}(t)$ to $\mathbf{y}(t+N-1)$ are taken into account, the model parameters can be obviously estimated with the least squares method. The following section reports the solution of this least squares problem by using of the well-known QR factorization [24–26]. It is summarized for better understanding as follows:

The $N \times dp+d$ data matrix is first constructed from available data:

$$\mathbf{K} = \begin{bmatrix} \mathbf{z}(t)^{\mathrm{T}} & \mathbf{y}(t)^{\mathrm{T}} \\ \mathbf{z}(t+1)^{\mathrm{T}} & \mathbf{y}(t+1)^{\mathrm{T}} \\ \cdots & \cdots \\ \mathbf{z}(t+N-1)^{\mathrm{T}} & \mathbf{y}(t+N-1)^{\mathrm{T}} \end{bmatrix} \tag{2}$$