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Time–frequency analysis of signals using support adaptive Hermite–Gaussian expansions

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ABSTRACT

Since Hermite–Gaussian (HG) functions provide an orthonormal basis with the most compact time–frequency supports (TFSs), they are ideally suited for time–frequency component analysis of finite energy signals. For a signal component whose TFS tightly fits into a circular region around the origin, HG function expansion provides optimal representation by using the fewest number of basis functions. However, for signal components whose TFS has a non-circular shape away from the origin, straight forward expansions require excessively large number of HGs resulting to noise fitting. Furthermore, for closely spaced signal components with non-circular TFSs, direct application of HG expansion cannot provide reliable estimates to the individual signal components. To alleviate these problems, by using expectation maximization (EM) iterations, we propose a fully automated pre-processing technique which identifies and transforms TFSs of individual signal components to circular regions centered around the origin so that reliable signal estimates for the signal components can be obtained. The HG expansion order for each signal component is determined by using a robust estimation technique. Then, the estimated components are post-processed to transform their TFSs back to their original positions. The proposed technique can be used to analyze signals with overlapping components as long as the overlapped supports of the components have an area smaller than the effective support of a Gaussian atom which has the smallest time-bandwidth product. It is shown that if the area of the overlap region is larger than this threshold, the components cannot be uniquely identified. Obtained results on the synthetic and real signals demonstrate the effectiveness for the proposed time–frequency analysis technique under severe noise cases.

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1. Introduction

Since Hermite–Gaussian (HG) functions constitute a natural basis for signals with compact time–frequency supports (TFSs), they have found applications in various fields of signal processing. In image processing, Hermite transform has been proposed for capturing local information [1]. Another image processing application is given in [2] for rotation of images. Also, in [3], HG functions are used for reconstruction of video frames. In telecommunications, highly localized pulse shapes both in time and frequency domains can be generated by using linear combinations of the HG functions [4]. As part of biomedical applications, representation of EEG and ECG signals in terms of HGs also have been proposed [5,6]. In [7], HG functions are used for characterization of the origins of vibrations in swallowing accelerometry signals. An electromagnetics application is reported in [8], where the time domain response of a three-dimensional conducting object excited by a compact TFS function is modeled by using HG expansions to obtain a fast extrapolator based on this expansion. Another electromagnetics application reported in [9], where a new method for evaluating distortion in multiple waveform sets in UWB communications has been proposed. Finally, as signal processing applications, HG functions are used for designing high resolution, multi-window time–frequency representation, where different order HGs are employed to realize multiple windows, and non-stationary spectrum estimation [10–13].

Single or multi-component signals with compact TFSs are frequently encountered in radar, sonar, seismic, acoustic, speech and biomedical signal processing applications [14–19]. Decomposition of such a signal into its components is an important application of time–frequency analysis [20]. For signals whose components have generalized time–bandwidth products of around 1, wavelet and chirplet based signal analysis techniques have been developed [21–23].

In this work, we are proposing a new signal analysis technique for signals whose components may have larger time-bandwidth

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products. Such signals are commonly employed in electronic warfare, including radar and sonar applications, because of their high resolution properties. Furthermore, biomedical signals including EEG and ECG have complicated time–frequency structures that significantly benefits from the proposed approach. The proposed technique makes use of adaptive HG basis expansion to estimate individual signal components. It is a well-known fact that HG functions form an orthonormal basis for the space of finite energy signals which are piecewise smooth in every finite interval [24]. What makes HGs special among other types of basis functions is their optimal localization properties in both time and frequency domains. For any circular TFS around the origin, HGs provide the highest energy concentration inside that region [25–27]. Therefore, if a signal component has a circular TFS around the origin, its representation by using the HG basis provides the optimal representation for a given number of representation order. However, if the signal component has a non-circular TFS positioned away from the origin, its HG representation is no longer optimal. Here, we propose an adaptive pre-processing stage where TFS of the signal component is transformed to a circular one centered around the origin so that it can be efficiently represented by HGs. The expansion order is estimated by a noise penalized costm function. Then, the desired representation is obtained by back transforming the identified signal component. For signals with multiple components that do not have overlapping TFSs, an EM based iterative procedure is proposed for joint analysis and expansion of individual signal components in HG basis.

The outline of the presentation is as follows. In Section 2, we give a brief review of HG functions and emphasize their fundamental properties. In Section 3, the proposed pre-processing stage is introduced. EM based iterative component estimation for analysis of multi-component signals and determination of optimal expansion orders are explained in Section 4. Results on synthetic and real signals are provided in Section 5. Conclusions are given in Section 6.

Note that, unless otherwise is stated, the integrals are computed in the $(-\infty,\infty)$ interval. Bold characters denote vectors, $(.)^H$ and $(.)^*$ are used for vector Hermitian and complex conjugation operations.

2. Review of Hermite–Gaussian functions

HG functions form a family of solutions to the following nonlinear differential equation:

$$f''(t) + 4\pi^2\left(\frac{2n+1}{2\pi} - t^2\right) f(t) = 0. \tag{1}$$

The nth order HG function $h_n(t)$ is related to the nth order Hermite polynomial $H_n(t)$ as

$$h_n(t) = \frac{2^{1/4}}{\sqrt{2^n n!}} H_n(\sqrt{2\pi}\,t)e^{-\pi t^2}, \tag{2}$$

where, with the initialization of $H_0(t) = 1$ and $H_1(t) = 2t$, $H_n(t)$ can be recursively obtained as

$$H_{n+1}(t) = 2tH_n(t) - 2nH_{n-1}(t). \tag{3}$$

Therefore, HG functions can also be computed recursively. A detailed discussion on HG functions and Hermite polynomials are available in [28] and [29], respectively. HG functions, of which the first four are shown in Fig. 1, form an orthonormal basis for the space of finite energy signals which are piecewise smooth in every finite $[-\tau,\tau]$ interval [24]. Hence, if $s(t)$ is in this space, it can be represented as

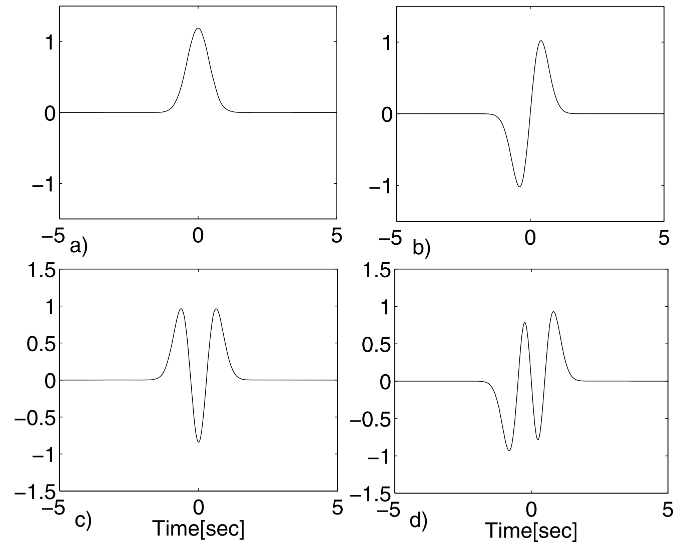

Fig. 1. The first four HG functions: (a) $h_0(t)$; (b) $h_1(t)$; (c) $h_2(t)$; (d) $h_3(t)$.

$$s(t) = \sum_{n=0}^{\infty} \alpha_n h_n(t), \tag{4}$$

where the expansion coefficients are

$$\alpha_n = \int h_n(t)s(t)\,dt. \tag{5}$$

Furthermore, HG functions are eigenvectors of the Fourier transformation [30]:

$$\mathrm{F}\{h_n(t)\} = \lambda_n h_n(t), \tag{6}$$

where F is the Fourier transform operator defined as $\mathrm{F}\{s(t)\} = \int s(t)e^{-j2\pi ft}\,dt$ and $\lambda_n = e^{-j\frac{\pi}{2}n}$ is its nth eigenvalue. Similarly, the fractional Fourier transform (FrFT) of order $-2 \leqslant a < 2$, also admits the HG functions as its eigenfunctions [31]:

$$\mathrm{F}^a\{h_n(t)\} = e^{-j\frac{\pi}{2}an}h_n(t), \tag{7}$$

where F^a is the FrFT operator of order a. Hence, FrFT of $s(t)$ can be obtained as:

$$\mathrm{F}^a\{s(t)\} = \sum_{n=0}^{\infty} \alpha_n e^{-j\frac{\pi}{2}an}h_n(t). \tag{8}$$

As seen from Eq. (7), the FrFT simply scales HGs. Thus, HG functions have circular support in the time–frequency plane. To demonstrate this fact, in Fig. 2, Wigner–Ville distribution of $h_0(t)$, $h_5(t)$, $h_{15}(t)$ and $h_{45}(t)$ are shown.

3. Support adaptive Hermite–Gaussian expansion

A piecewise smooth signal $s(t)$ can be approximated by using the following Lth order HG expansion:

$$\tilde{s}^{(L)}(t) = \sum_{n=0}^{L} \alpha_n h_n(t), \tag{9}$$

with its corresponding normalized approximation error:

$$e^{(L)} = \int \left|s(t) - \tilde{s}^{(L)}(t)\right|^2 dt \Big/ \int \left|s(t)\right|^2 dt, \tag{10}$$

where α_n are obtained as in (5). Since the basis functions are orthonormal, in the absence of noise, by increasing the expansion order L, the approximation error can be decreased. However, for

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