Digital Signal Processing 22 (2012) 1073–1084

Contents lists available at SciVerse ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

# Identification of sparse impulse responses – design and implementation using the partial Haar block wavelet transform

Celso H.H. Ribas [a,1], José C.M. Bermudez [b,*,2], Neil J. Bershad [c]

[a] Anatel – National Agency of Telecommunications, Brazil
[b] Dept. of Electrical Engineering, Federal University of Santa Catarina, Trindade, Florianópolis, SC, 88040-970, Brazil
[c] Dept. of Electrical Engineering and Computer Science, University of California, Irvine, CA, USA

ARTICLE INFO

*Article history:*
Available online 15 June 2012

*Keywords:*
Adaptive filtering
System identification
Sparse

ABSTRACT

This paper proposes an implementation for identifying sparse impulse responses. The new scheme follows the approach in which the location of the channel response peak is estimated in the wavelet domain. A short time-domain adaptive filter is then located about the estimated peak to identify the sparse response. The primary purpose of this paper is to present an efficient design of such a system. The use of a new block wavelet transform results in up to 70% less computational complexity and improved peak detection, as compared to previous solutions. A new robust time-domain adaptive filtering location and update scheme is also proposed that significantly reduces the occurrence of jitter problems and leads to improved residual mean-square error performance. The behavior of the transform-domain adaptive filter is analyzed, the Wiener solution is determined, and an accurate analytical model is obtained for the mean-square deviation of the adaptive coefficients. Monte Carlo simulations show excellent echo cancellation performance for typical ITU-T echo channels.



## 1. Introduction

Sparse impulse responses are encountered in many applications [1]. One important example is network echo cancellation. Bulk delays in echo paths are often much longer than the actual echo path impulse response. Typically, the bulk delay can be on the order of 128 ms [2]. Measurements in North America have shown that most dispersion times (active impulse response length) are between 5–7 ms [3]. Similar results were found in Europe. Thus, most of the adaptive coefficients in a 128-ms delay line (1024 coefficients for an 8 kHz sampling rate) will be zero. Hence, conventional FIR adaptive filtering becomes inefficient, as long adaptive filters are both slow to adapt and have noisy weights [4]. Several algorithms have been proposed to exploit sparsity for improving sparse response identification efficiency. A good overview of these techniques can be found in [5]. One successful approach is to locate the significant (active) samples of the unknown response [6–8,2,9].

The solution proposed in [6] is able to identify responses with an unknown number of dispersive regions. However, it requires a number of preset parameters, which makes optimal design difficult. A least-squares (LS) solution based on active tap detection is used with the NLMS algorithm in [7]. This scheme improves previous solutions by the first author of [7] and others (see references in [7]) for correlated input signals. A forgetting factor is also included for tracking purposes. The computational complexity is slightly greater than NLMS if two look-up tables are used. It has been reported that the LS-based detection may fail for impulse responses with large dynamic ranges [5]. The work in [8] exploits the wavelet transform (WT) time hierarchy in sparse system identification. The Haar-basis (HB) algorithm in [8] operates using a control scale. All weights in this scale are adapted at each adaptation interval (AI). Converged control weights larger than a detection threshold activate the weights in the same time hierarchy at the other scales after each AI. A new AI then begins with all active and control weights adapting. This approach works over the entire impulse response and can be used to identify sparse responses with more than one dispersive region. The weight activations are based on a fixed control scale. Thus, the algorithm may fail for sparse responses that are not rich enough in frequency content [5]. Examples can be found among typical echo path responses [3,9]. Two new algorithms based on the wavelet packet transform have been recently proposed in [9,10] to identify sparse impulse responses with any number of dispersive regions. The solutions in [9,10] adaptively design the wavelet packet transform to match the characteristics of the unknown response. Such solutions lead to a reduced number of active weights as compared to [8]. This

* Corresponding author.
*E-mail addresses:* celsoribas@gmail.com (C.H.H. Ribas), j.bermudez@ieee.org (J.C.M. Bermudez), bershad@ece.uci.edu (N.J. Bershad).

[1] Celso H.H. Ribas is an independent consultant.

[2] This work was partly supported by CNPq under grants No. 305377/2009-4 and 473123/2009-6.

1051-2004/$ – see front matter 
http://dx.doi.org/10.1016/j.dsp.2012.06.004

is largely due to the flexibility of the wavelet packet transform to adjust to the frequency content of the unknown response.

Simpler solutions can be very effective when the unknown response has a single dispersive region (as in typical network echo cancellation [3]) or more than one dispersive region with small intervals between any two consecutive regions. Two short adaptive filters are operated sequentially in [2]. One adaptive filter operates in a partial Haar transform domain to estimate the location of the peak of the unknown response. The second filter is a short time-domain adaptive filter centered about the estimated peak location, which adapts to identify the nonzero portion of the echo path impulse response. The algorithm uses the time hierarchy of the WT to position the time-domain filter. Hence, two short adaptive filters can be used instead of one long adaptive filter. This results in faster overall convergence and reduced computational complexity and storage.

The solution in [2] represents an effective way to identify sparse network echo responses. However, its performance is influenced by important design issues: (1) computational complexity for implementing the wavelet transform, (2) centering of the time-domain adaptive filter about the estimated peak, and (3) bulk delay tracking leads to a jitter problem as the peak estimate changes. These three design issues were not studied in [2]. Although the basic system studied here is the same as in [2], the implementation is quite different and leads to significant performance improvement.

This paper presents a scheme for identifying sparse impulse responses that is based on the approach [2] (part of this work has been presented in [11]). However, the sparse response bulk delay is estimated in the transform domain using a block-processing scheme. This approach provides a time-shifting property to the transformed input signal vector that yields computational savings of more than 85% (compared to [2]) for the peak detection. The new block wavelet transform also locates the impulse response peak more clearly. Thus, the peak detection is also improved in the transform domain. Finally, a new and more robust approach is presented for the time-domain adaptive filter window location and update. This new approach significantly reduces the jitter problem encountered in [2]. An analytical model is developed for the mean-square deviation of the transform-domain adaptive filter coefficients. Monte Carlo simulation examples show excellent results for network echo cancellation and the accuracy of the analytical model for typical echo impulse responses from the ITU-T recommendation G.168 [3].

## 2. The partial Haar transform

The Haar wavelet transformation of a vector of length $2^r$ is formed by pre-multiplication with a $2^r \times 2^r$ matrix $\boldsymbol{H}$ [12]. The partial Haar transformation in [2] uses only a single wavelet scale, say the $m$th. The corresponding matrix $\boldsymbol{H}_m$ is a $2^{r-m} \times 2^r$ submatrix of $\boldsymbol{H}$ whose elements are defined as

$$\boldsymbol{H}_m(i,j) = \psi_m\big[j-(i-1)2^m-1\big],\quad i=1,\ldots,2^{r-m},\ j=1,\ldots,2^r, \tag{1}$$

where

$$\psi_m(\ell) = \begin{cases} 2^{-\frac{m}{2}}, & 0 \leqslant \ell \leqslant 2^{m-1}-1, \\ -2^{-\frac{m}{2}}, & 2^{m-1} \leqslant \ell \leqslant 2^m-1, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

As an example, for $r=3$ and $m=2$, the partial Haar matrix is given by

$$\boldsymbol{H}_2 = \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & -0.5 & -0.5 \end{bmatrix}. \tag{3}$$

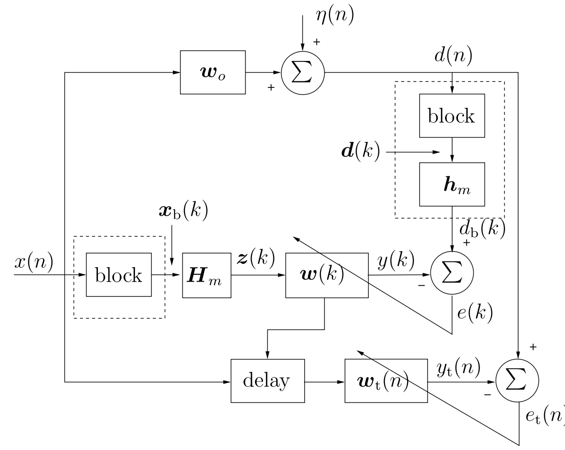


**Fig. 1.** Block diagram of the proposed implementation.

The choice of $m$ determines the number of coefficients of the partial Haar transformation and thus the length of the transform-domain adaptive filter. The $m$th scale partial-transform domain adaptive filter uses only $2^{r-m}$ coefficients for a response of length $2^r$. This adaptive filter cannot exactly model the length-$2^r$ impulse response. However, this is not required here since the transform-domain adaptive filter is only used to estimate the location of the channel impulse response peak.

Note that every row of $\boldsymbol{H}_m$ is composed of the same unity-norm nonzero basis vector $\boldsymbol{h}_m^T$ and additional zeros, where

$$\boldsymbol{h}_m^T = \big[\psi_m(0),\ldots,\psi_m\big(2^m-1\big)\big] \tag{4}$$

is the wavelet in the $m$th partial. For example, $\boldsymbol{h}_2^T = [0.5, 0.5, -0.5, -0.5]$ in (3). Moreover, all coefficients have the same magnitude. These properties can be used to reduce implementation complexity.

## 3. The Partial Haar DWT LMS algorithm

### 3.1. The new scheme

The computational complexity of [2] arises primarily from computing the partial Haar transform vector of the input at each time instant. This is because the transformed vector does not have a shift structure, which is the structure observed in vectors that arise from a tapped delay line implementation. Thus, the entire transformed vector must be evaluated at each iteration. Only the most recent transformed output sample would need to be evaluated at each iteration if the transformed vector had a shift structure. This property holds if the time-domain signals are processed in blocks of length $2^m$ when the $m$th partial transform is used. For example, an input vector of length 8 is composed of two subvectors of length 4 in (3). Each subvector is multiplied by the same basis vector $\boldsymbol{h}_m^T$ to generate the two samples of the transformed vector. Only one new transform sample must be evaluated at each iteration if the input vector samples advance in blocks of length 4 ($2^m$ for $m=2$). The transform vector will then have the shift structure, and the adaptive weight vector remains constant for the duration of each block.

Fig. 1 shows a block diagram of the proposed implementation. Here, $n$ is the time sample index, $k$ is the block index, $x(n)$ is the input signal, $\boldsymbol{w}_{\rm o}$ is the unknown parameter vector, $\eta(n)$ is the additive noise, $d(n)$ is the desired signal, $\boldsymbol{w}(k)$ is the wavelet-domain adaptive weight vector, and $\boldsymbol{w}_{\rm t}(n)$ is the time-domain adaptive weight vector. The input vector $\boldsymbol{z}(k)$ to the wavelet-domain adaptive filter has the shift structure. The top element of $\boldsymbol{z}(k)$ and the