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Structured Lyapunov functions for synchronization of identical affine-in-control agents—Unified approach ☆

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Abstract

This paper brings structured Lyapunov functions guaranteeing cooperative state synchronization of identical agents. Versatile synchronizing region methods for identical linear systems motivate the structure of proposed Lyapunov functions. The obtained structured functions are applied to cooperative synchronization problems for affine-in-control nonlinear agents. For irreducible graphs a virtual leader is used to analyze synchronization. For reducible graphs a combination of cooperative tracking and irreducible graph cooperative synchronization is used to address cooperative dynamics by Lyapunov methods. This provides a connection between the synchronizing region analysis, incremental stability and Lyapunov cooperative stability conditions. A class of affine-in-control systems is singled out based on their contraction properties that allow for cooperative stability *via* the presented Lyapunov designs.
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1. Introduction

The last two decades have witnessed an increasing interest in multi-agent networked cooperative systems [1,5,10,11,15–17,21,26]. Early work [5,15–17,21] refers to *consensus* without a leader. We term this the *cooperative regulator* problem. There the asymptotic

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consensus state depends on precise initial conditions of an entire system. By adding a command generator leader that pins to a group of agents one can have *synchronization* to the leader's reference trajectory for all initial conditions; this is termed *pinning control* [2,8,24,26,27,31]. There, pinning to all root nodes of a spanning forest is necessary for synchronization [31]. We call this the *cooperative tracker problem*.

Necessary and sufficient conditions for synchronization are given by master stability functions [20,24,33] and the related concept of synchronizing regions [4,24–26]. This guarantees local stability. For linear systems, however, local and global stability coincide; hence the synchronizing region approach yields global results. Synchronizing region and pinning control papers often *a priori* assume inner coupling functions having special properties [2,4,12,24,26], thereby disregarding the controllability properties inherent to single-agents.

Global results for nonlinear systems are generally obtained by Lyapunov methods [27,30,32] or contraction analysis, *i.e.* incremental stability [13,18,22]. Especially interesting for interconnected systems are the results involving incremental stability [13,22,33,34] and incremental passivity [19]. Often Lyapunov methods in the literature either assume certain *a priori* forms of the drift dynamics and inner coupling functions or restrict their considerations to undirected or balanced graphs. For example, special drift dynamics (QUAD) is assumed in [2,12] to guarantee a quadratic bound on the pertaining contribution to dissipation, and the distributed control is taken as all-state direct feedback in [2,23,31,32] to completely dominate the bounded effect of the drift dynamics. Other special properties of the inner coupling function are assumed in [12,26,31,32,43]; *e.g.* diagonal form [12,26,43], positive definiteness [32], or positive definite contribution to dissipation [31]. In [40] vector double integrator agents are considered and the underlying graph topologies, although allowed to be switching, are assumed undirected. The approach in [40] relies on joint Lyapunov functions. Similarly [41], although considering a different notion of consensus, also assumes undirected graphs and double integrator agents. Consistent with restricting attention to double integrators, the leader's reference signal in both [41,42] is constant. Developments of [41] use Laplacian potentials for undirected graphs [27]. More general Laplacian potentials, in part, motivate the approach of this paper as well. The contraction approach [13,18,32,33,37], in contrast, occupies middle ground between the local linearization results of synchronizing regions and global Lyapunov conclusions, in the sense that linearized dynamics is used but stability requirements hold uniformly, implying global results [13,36]. However, in [32,33] also *a priori* assumptions on inner coupling functions are made without considering how to guarantee them for a given system.

Any *a priori* conditions on inner coupling functions disregard the given controllability properties of single-agents. Realistic systems are characterized by their controllability structure and possibly the constraints of output-feedback. This restricts the feasible distributed controls, and must be accounted for in the control design. Furthermore, *a priori* choices of inner coupling matrix and drift dynamics appear somewhat artificial in light of the fact that it is possible, within the synchronizing region approach, to obtain the required properties by design. For example, in [8,11,25,27] the inner coupling matrix is designed by considering the given single-agent systems. The resulting feedback interplays with the drift dynamics to guarantee cooperative stability. Namely, for linear time-invariant (LTI) agent synchronization [11,25,27] use single-agent optimal feedback derived from algebraic Riccati equations (ARE). Such control guarantees an unbounded right-half plane synchronizing region. Hence, state synchronization is achieved under mild requirements on directed communication topology, utilizing given stabilizability properties of individual agents rather than imposing them *a priori* by assumptions. Apart from accounting for the controllability relations, synchronizing region approach [20,25,26,28] also treats cases of

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