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New state transformations of time-delay systems with multiple delays and their applications to state observer design

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Abstract

This paper addresses the problem on how to systematically compute state transformations of a general class of time-delay systems with multiple time delays in the state and output vectors, and applications of these state transformations to state observer design. First, a forward state transformation problem is studied. For this, we present a two-stage coordinate transformation method to transform any given time-delay system into an observable canonical form where time delays appear in the input and output vectors, but not in the state vector. The significance of such a coordinate transformation is that a Luenberger-type state observer can be easily designed in the new coordinate system. Then, a backward state transformation problem is studied which allows us to reconstruct the original state vector of the system. Therefore, by using both the forward and the backward state transformations, state observers for time-delay systems can be systematically designed. Our approach based on the new state transformations enables the design of state observers of a more general class of time-delay systems than existing works in the literature. Conditions for the existence of the state transformations and an algorithm for computing them are provided in this paper. Numerical examples and simulation results are given to illustrate the effectiveness of our proposed design method.

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1. Introduction

Since time-delay is often a source of instability and performance degradation in many practical systems, considerable attention has been paid to the problem of stability analysis, controller synthesis and state estimation of time-delay systems over the past years (see, for example [1–37], and the references therein). In particular, the problem of designing state observers has many important applications in the realization of state feedback control, system supervision, fault diagnosis of dynamic processes, and general control and diagnosis issues from available information [7–12]. In the literature, various methods have been proposed to design state observers to estimate the state vector of time-delay systems (see, for example, [7,13–22]). Most of the design methods available in the literature [7,13–22]) are primarily for systems with a single time delay in the state vector. However, it is necessary to consider the state estimation problem of time-delay systems with multiple time delays in the state vector and also in the output vector (for brevity, we refer to such systems as MDSO systems). Since it is often the case that interconnected time-delay systems have multiple time delays in the state vector [23], and systems which are network-controlled or network-connected usually have their output measurements subjected to time delays. Therefore, it is important to address the state estimation problem of MDSO systems.

With regard to MDSO systems, some interesting results on H_∞ filtering have been reported (see, for example [24–33], and the references therein). Nevertheless, research into state observer design of MDSO systems has, so far, received lesser attention. In [36], the authors proposed a state transformation approach to design state observers for MDSO systems. The novel idea of their observer design method is to find a coordinate state transformation such that in the new coordinates, all the time-delay terms in the system description are associated with the output only and thus a state observer can be easily designed. However, as recently reported in [37], the existence conditions of the state transformations reported in [36] are quite restrictive. Thus, when the existence conditions are not fulfilled, the state transformations cannot be computed, and as a result, the design of state observers for MDSO systems cannot be carried out. To overcome this drawback, in [37], the authors proposed a new algebraic and systematic method for computing state transformations of time-delay systems. It was reported that of the three possible versions of the original state vector, namely, instantaneous, delayed, and a mixed of instantaneous and delayed, a state observer which estimates one of these versions can be designed by using their state transformations. Nevertheless, due to the complexity and the difficulty of the problem, the authors [37] only addressed the case where there is one time-delay in the state vector and the number of the outputs in the system is restricted to two only. So far, the state transformation method reported in [37] has not been extended to general case with multiple outputs, and multiple time delays in both the state vector and the output vector. While, as we have explained in the aforementioned paragraph, it is necessary and important to address the state observer design of MDSO systems.

In this paper, motivated by the work of [36,37], we consider the problem of computing, in a systematic manner, novel state transformations of a class of MDSO systems. Firstly, the forward state transformations are computed to transform the time-delay system into an observable canonical form where time delays appear only in the input and output vectors. The approach taken to derive the forward state transformations is algebraic and we provide a systematic procedure for computing the forward state transformations. Once a given time-delay system is transformed into an observable form, any Luenberger-type state observer can be easily designed. Then, we discuss the backward state transformation which enables the original state vector of the