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Laguerre functions approximation for model reduction of second order time-delay systems ☆

Xiaolong Wang[a,*], Yaolin Jiang[b], Xu Kong[c]

[a]School of Science, Northwestern Polytechnical University, Xi'an 710072, PR China
[b]Department of Mathematics, Xinjiang University, Urumqi, 830046, PR China
[c]School of Mathematical Sciences, Liaocheng University, Liaocheng 252000, PR China

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Abstract

This paper presents a Laguerre-based model reduction method for time-delay systems, which naturally retains the second-order-structure of original systems. We expand time-delay systems under the Laguerre function basis and show that Laguerre coefficients satisfy a special linear system and therefore can be calculated iteratively. Then reduced models are produced using projection methods, where projection matrices are defined in terms of the derived Laguerre coefficients, while properties on the preservation of Laguerre coefficients are proved elegantly based on the linear system corresponding to Laguerre coefficients. We also discuss the practical implementation of the approach. A high order Arnoldi procedure with memory saving is provided which leads to an efficient execution of our approach in practice. Numerical examples are simulated to illustrate the feasibility and efficiency of our approach.

1. Introduction

Many physical systems and processes are modeled mathematically in terms of higher order differential equations. In large scale settings, the simulation of such models is extremely time and storage consuming. Model reduction approximates a high order system with a lower order one so

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*Corresponding author.
E-mail address: xlwangmath@gmail.com (X. Wang).

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as to significantly reduce the computational cost and the simulation time. Model reduction methods for standard linear time-invariable (LTI) systems have been studied extensively in the literature, mainly including moment-matching methods and balanced truncation methods [1].

Time-delay systems belong to a class of functional differential equations, which arise naturally in the phenomena involving events occurring in a non-instantaneous manner [2]. In networked control systems, for example, the so-called network-induced time-delays happen frequently due to the extensive data exchange among the device layer and operational layer channel, and it is advisable to reflect such effects in the overall description of the industrial processes [3–6]. Several approaches are available for model reduction of systems with time delays. In [7,8], time-delay systems are approximated using rational interpolation. Some researchers simply approximate delay terms appearing in the systems by a Padé approximation or a Taylor series expansion. This actually leads to a high order system free of time delays and thereby the standard model reduction approaches can be applied directly [9,10]. Further, an extension of the moment matching method is proposed in [11] relying on an Arnoldi process, where by the spectral discretization a time-delay system is reformulated equivalently as an infinite-dimensional linear problem and then reduced by building appropriate Krylov spaces for the linear infinite-dimensional problem. This approach can be implemented efficiently if a two-level orthogonalization Arnoldi process is used in the definition of projection matrices, instead of the classic Arnoldi process [12]. However, the surrogate system free of delays ignores the effects which are adequately represented by time delay terms and is not a general alternative.

Another way of performing reduction for time-delay systems leads to the problem of H_∞ model reduction, which aims at the error norm between the original system and the reduced model less than a given tolerance [13–16]. Typically, some sufficient conditions for the desired H_∞ performance are first derived, and then reduced models are extracted by solving a set of linear matrix inequalities (LMIs). However, as the resolution of these LMIs present enormous computational challenges even for medium scale problems, the H_∞ model reduction is not applicable in the large scale settings. Note that the strength of H_∞ reduction lies in its universal applicability, which allows to address the problem in a unified framework no matter time delays are constant or time-varying [17–20]. This approach has also been extended to the time-delay systems with Markovian jumping parameters, as well as the problems of fault detection and reliable control [21–23]. Besides, there are methods based on the dominance of poles and balanced truncation approach for model reduction of time-delay systems [24–27], but the resulting reduced models are not guaranteed to attain the nice properties in the linear case. In order to enable an efficient execution for model reduction of time-delay systems and to retain the second-order-structure of systems, in this paper we generalize Laguerre-based reduction methods to the second order time-delay systems. As Laguerre-based methods rely on the system transfer function, we restrict ourselves to the linear systems with constant time delays. Nevertheless, the theoretic results in this paper have the potential to apply to switched systems with constant time delays, as a switched system boils down to our situations when a switching signal is specified [18,20]. For the general approach on Laguerre-based reduction, we refer to [28–33] and references therein.

Our approach is based on the Laguerre expansion of time-delay systems in the frequency domain. Unlike the current schemes using the truncated Taylor expansion, we adopt the complete Laguerre expansion for time delays and show that Laguerre coefficients satisfy a linear system and can be calculated iteratively. Then, a second order reduced model with time delays is generated in the framework of projection methods. The property on the preservation of Laguerre coefficients is proved based on the linear system Laguerre coefficients satisfy, leading to a more compact proof. This is in contrast to the current techniques where such a kind of properties is