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An evidence clustering DSmT approximate reasoning method for more than two sources

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ABSTRACT

Due to the huge computation complexity of Dezert–Smarandache Theory (DSmT), its applications especially for multi-source (more than two sources) complex fusion problems have been limited. To get high similar approximate reasoning results with Proportional Conflict Redistribution 6 (PCR6) rule in DSmT framework (DSmT + PCR6) and remain less computation complexity, an Evidence Clustering DSmT approximate reasoning method for more than two sources is proposed. Firstly, the focal elements of multi evidences are clustered to two sets by their mass assignments respectively. Secondly, the convex approximate fusion results are obtained by the new DSmT approximate formula for more than two sources. Thirdly, the final approximate fusion results by the method in this paper are obtained by the normalization step. Analysis of computation complexity show that the method in this paper cost much less computation complexity than DSmT + PCR6. The simulation experiments show that the method in this paper can get very similar approximate fusion results and need much less computing time than DSmT + PCR6, especially, when the numbers of sources and focal elements are large, the superiorities of the method are remarkable.

1. Introduction

With the rapid development of science and technology, sensor types become diverse. To solve the more and more complex practical problems, information fusion from different multi sources has been drawn great attention by scholars in recent years [1–12]. Due to the influence of noise, uncertain signal and information processing has become an important research direction in the field of information fusion. Belief function theory (also called evidence theory) has played a key role in uncertain and even conflict information processing. As a traditional evidence theory, Dempster–Shafer theory (DST) [13,14] is a general applied information fusion method. However, DSmT, jointly proposed by Dezert and Smarandache [12], beyonds the exclusiveness limitation of DST and especially in highly conflict information cases, it can obtain more accurate fusion results than DST. Recently, DSmT has many successful applications, such as, Map Reconstruction of Robot [15], Decision Making Support [16,17], Target Type Tracking [18], Image Processing [19], Sonar Imagery [20], Data Classification [21–25], Clustering [26–28], and so on. Besides, neutrosopic theory [29–31] proposed by Smarandache is a novel effective uncertain information processing method. However, the main problem of DSmT is that when the number of sources and focal elements increases, the computation complexity of PCR5 or PCR6 in DSmT framework increases exponentially [32].

There were some important methods for reducing the computation complexity of the combination algorithms in DSmT framework since this problem can be treated in different ways: 1) reducing the number of focal elements [33–36], 2) reducing the number of combined sources [37], 3) reducing both the number of focal elements and the number of combined sources [38]. Applied mathematics has drawn attention by many scholars [39,40]. Particularly, the very recent Evidence Clustering DSmT Approximate Reasoning Method based on Convex Function Analysis [11] proposed by Guo, He, et al can get very similar approximate fusion results with PCR5 in DSmT framework and cost little computation complexity. Nevertheless, the approximate method in [11] is only for two sources and it is not associative in the fusion of multiple (more than 2) sources of evidences.

For reducing the huge computation complexity of Dezert–Smarandache Theory (DSmT) for multi-source (more than two sources) complex fusion problems and get more similar approximate fusion results with PCR6 in DSmT framework (DSmT + PCR6), a DSmT Approximate Reasoning Method for More than Two Sources is proposed in this paper. In Section 2, the basics knowledge on DST, DSmT + PCR6 and evidential distance theory are in-

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troduced briefly. In Section 3, a new form of PCR6 formula is given and the mathematical analysis of the new form is conducted. Then a new DSmT approximate reasoning method for more than two sources is proposed. Finally, Analysis of computation complexity shows that the method in this paper costs much less computation complexity than DSmT + PCR6. In Section 4, the simulation experiments show that the method in this paper can get very similar approximate fusion results and need much less computing time than DSmT + PCR6, especially, when the number of sources and focal elements is large, the superiorities of the method are remarkable. In Section 5, the conclusions are given.

2. Basic knowledge

2.1. Dempster–Shafer theory (DST)

A discernment frame based on the Shafer's model is defined as $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$ which contains n exclusive elements. The mass assignments of evidences defined over the power-set 2^Θ is defined by

$$m(X_i) : 2^\Theta \to [0,1], \quad X_i \in 2^\Theta \tag{1}$$

If $m(X_i) > 0$, X_i is called the focal element. m_i denotes the mass assignments of the ith source of evidence. The Dempster's combination rule is given by [13,14]

$$m_{DS}(X) = \frac{\sum_{X_i \cap X_j = X, i \neq j} m_1(X_i) \cdot m_2(X_j)}{1-K}, \quad \forall X \subseteq \Theta \tag{2}$$

$$K = \sum_{\substack{X_i, X_j \subseteq \Theta, i \neq j \\ X_i \cap X_j = \emptyset}} m_1(X_i) \cdot m_2(X_j) \tag{3}$$

where K denotes the conflict beliefs of evidences. However, when the conflict beliefs of evidences are high, the fusion results of Dempster's combination rule are usually very unreasonable. For this reason, many combination rules were developed, especially, Proportional Conflict Redistribution1–6 (PCR1–6) rules in DSmT framework have many advantages and successful applications [12].

2.2. Dezert–Smarandache theory (DSmT)

The discernment frame of DSmT based on the hyper-power set abandons the exclusiveness limitation of DST. The hyper-power set denoted by D^Θ admits the intersection of the elements. For example, if there are two elements in discernment framework $\Theta = \{\theta_1, \theta_2\}$, the hyper-power set is $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2, \theta_1 \cap \theta_2\}$. The mass assignments of evidences defined over the hyper-power set D^Θ is defined by

$$m(X_i) : D^\Theta \to [0,1], \quad X_i \in D^\Theta \tag{4}$$

The Proportional Conflict Redistribution (PCR) rules [41,42] are the combination rules in DSmT framework. PCR rules have PCR1-6 rules and the difference of them is that the proportional redistribution way of the conflict beliefs. PCR1–5 rules are applied for the combination of two sources and among these rules, PCR5 is considered as the most precise mathematical method. PCR6 rule is usually applied for more than two sources fusion problems.

PCR5 rule for 2 sources is introduced as follows [41,42]

$$m_{1\oplus2}(X_i) = \sum_{\substack{Y,Z \in G^\Theta \text{ and } Y,Z \neq \emptyset \\ Y \cap Z = X_i}} m_1(Y) \cdot m_2(Z) \tag{5}$$

$$m_{PCR5}(X_i) = \begin{cases} m_{1\oplus2}(X_i) \\ \quad + \sum_{\substack{X_j \in G^\Theta \text{ and } i \neq j \\ X_i \cap X_j = \emptyset}} \left[\frac{m_1(X_i)^2 \cdot m_2(X_j)}{m_1(X_i)+m_2(X_j)} + \frac{m_2(X_i)^2 \cdot m_1(X_j)}{m_2(X_i)+m_1(X_j)}\right] \\ \quad X_i \in G^\Theta \quad \text{and} \quad X_i \neq \emptyset \\ 0, \quad X_i = \emptyset \end{cases} \tag{6}$$

where G^Θ can been seen as the power set 2^Θ, the hyper-power set D^Θ and the super-power set S^Θ, if discernment of the fusion problem satisfies the Shafer's model, the hybrid DSm model, and the minimal refinement Θ^{ref} of Θ respectively and where all denominators are more than zero and the fraction is discarded when the denominator of it is zero [41,42].

This paper is mainly for more than two sources fusion. PCR6 rule for more than 2 sources is introduced as follows

$$m_{1\oplus2\oplus\cdots\oplus s}(X) = \sum_{\substack{Y_1, Y_2, \cdots, Y_s \in G^\Theta \text{ and } Y_1, Y_2, \cdots, Y_s \neq \emptyset \\ Y_1 \cap Y_2 \cap \cdots \cap Y_s = X}} m_1(Y_1) \times m_2(Y_2) \times \cdots \times m_s(Y_s) \tag{7}$$

$$m_{ConflictTransfer}(X) = \sum_{\substack{Z_1, Z_2, \cdots, Z_{s-1} \in G^\Theta \\ Z_i \neq X, i \in \{1,2,\cdots,s-1\} \\ (\bigcap_{i=1}^{s-1} Z_i) \cap X = \emptyset}} \sum_{k=1}^{s-1} \sum_{(i_1, i_2, \cdots, i_s) \in P(1,2,\cdots,s)} [m_{i_1}(X) + m_{i_2}(X) + \cdots + m_{i_k}(X)] \cdot \left[\frac{m_{i_1}(X) \times m_{i_2}(X) \times \cdots \times m_{i_k}(X) \times m_{i_{k+1}}(Z_1) \times \cdots \times m_{i_{k+1}}(Z_{s-k})}{m_{i_1}(X) + m_{i_2}(X) + \cdots + m_{i_k}(X) + m_{i_{k+1}}(Z_1) + \cdots + m_{i_{k+1}}(Z_{s-k})}\right] \tag{8}$$

$$m_{PCR6}(X) = m_{1\oplus2\oplus\cdots\oplus s}(X) + m_{ConflictTransfer}(X), \quad X \in G^\Theta \quad \text{and} \quad X \neq \emptyset \tag{9}$$

where G^Θ denotes the general power set which can be seen as the same as 2^Θ, D^Θ or the super-power set S^Θ in different cases; and $P(1, 2, \cdots, s)$ denotes the set of all permutations of the elements. Equation (7) denotes that the combination products of the intersections of the mass assignments. Equation (8) denotes that the proportional redistribution of the conflict beliefs of mass assignments.

Assume that $s = 2$, PCR6 rule is given by

$$m_{1\oplus2}(X) = \sum_{\substack{Y_1, Y_2 \in G^\Theta \text{ and } Y_1, Y_2 \neq \emptyset \\ Y_1 \cap Y_2 = X}} m_1(Y_1) \times m_2(Y_2) \tag{10}$$

$$m_{ConflictTransfer}(X) = m_{i_1}(X) \cdot \left[\frac{m_{i_1}(X) \times m_{i_2}(Z_2)}{m_{i_1}(X) + m_{i_2}(Z_2)}\right] + m_{i_2}(X) \cdot \left[\frac{m_{i_2}(X) \times m_{i_1}(Z_1)}{m_{i_2}(X) + m_{i_1}(Z_1)}\right] \tag{11}$$

$$m_{PCR6}(X) = \sum_{\substack{Y_1, Y_2 \in G^\Theta \text{ and } Y_1, Y_2 \neq \emptyset \\ Y_1 \cap Y_2 = X}} m_1(Y_1) \times m_2(Y_2) + m_{i_1}(X) \cdot \left[\frac{m_{i_1}(X) \times m_{i_2}(Z_2)}{m_{i_1}(X) + m_{i_2}(Z_2)}\right] + m_{i_2}(X) \cdot \left[\frac{m_{i_2}(X) \times m_{i_1}(Z_1)}{m_{i_2}(X) + m_{i_1}(Z_1)}\right], \quad X \in G^\Theta \quad \text{and} \quad X \neq \emptyset \tag{12}$$