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Circularly-symmetric complex normal ratio distribution for scalar transmissibility functions. Part II: Probabilistic model and validation

Wang-Ji Yan, Wei-Xin Ren *

Department of Civil Engineering, Hefei University of Technology, Hefei, Anhui 23009, China

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ABSTRACT

In Part I of this study, some new theorems, corollaries and lemmas on circularly-symmetric complex normal ratio distribution have been mathematically proved. This part II paper is dedicated to providing a rigorous treatment of statistical properties of raw scalar transmissibility functions at an arbitrary frequency line. On the basis of statistics of raw FFT coefficients and circularly-symmetric complex normal ratio distribution, explicit closed-form probabilistic models are established for both multivariate and univariate scalar transmissibility functions. Also, remarks on the independence of transmissibility functions at different frequency lines and the shape of the probability density function (PDF) of univariate case are presented. The statistical structures of probabilistic models are concise, compact and easy-implemented with a low computational effort. They hold for general stationary vector processes, either Gaussian stochastic processes or non-Gaussian stochastic processes. The accuracy of proposed models is verified using numerical example as well as field test data of a high-rise building and a long-span cable-stayed bridge. This study yields new insights into the qualitative analysis of the uncertainty of scalar transmissibility functions, which paves the way for developing new statistical methodologies for modal analysis, model updating or damage detection using responses only without input information.

1. Introduction

Due to its clear physical interpretation in characterizing the dynamics of a system, scalar transmissibility functions arise in many related fields such as structural damage detection [1–13], modal analysis [14–22], model updating [23,24] and structural design [25,26], etc. A scalar transmissibility function at an arbitrary frequency line is a random variable with certain statistical distribution in essence [13,27]. A brute-force MCS, extensively employed to perform uncertainty quantification due to its generality and stability [28,29], may be impractical in the context of stochastic signal processing due to its complicated features. There is a profound common viewpoint that probability density function (PDF) is the most clear, fundamental and sufficient probabilistic descriptions of the uncertainty behavior [30]. Therefore, the primary focus of this study is to characterize the PDF of raw scalar transmissibility functions analytically.

* Corresponding author.
E-mail address: renwx@hfut.edu.cn (W.-X. Ren).

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It has been proved that the PDF of raw FFT coefficients without artificial averaging or smoothing at a specific frequency line approximately follows multivariate circularly-symmetric complex normal distribution for stationary stochastic vector process as the duration of the analyzed data tends to infinity [31,32]. Based on the statistics of FFT coefficients, one can further conclude that PSD matrix asymptotically follows the Wishart distribution [32] and the trace of a PSD matrix can be well approximated by normal distribution under certain conditions [33]. The ingenious discovery in [31,32] has led to the now burgeoning interest in Bayesian operational modal analysis [33–39] as well as Bayesian model updating in the frequency domain [40,41]. As is pointed in Part I of this study [42], given a specific reference output, multivariate correlated scalar transmissibility functions can be formulated for a system with multiple outputs. A scalar transmissibility function is viewed as a univariate circularly-symmetric complex normal ratio random variable, while a raw transmissibility vector follows multivariate circularly-symmetric complex normal ratio distribution.

A rigorous proof of new theorems on multivariate circularly-symmetric complex normal ratio random variables as well as some corollaries and lemmas were presented in Part I [42]. The mathematical basis is then applied in developing new probabilistic model for raw scalar transmissibility functions in this paper by incorporating the statistics of raw FFT coefficients. Section 2 of this article revisits the theorems and corollaries on circularly-symmetric complex normal ratio distribution for the convenience of readership. Section 3 is the core of this paper, in which the statistics of raw FFT coefficients are interpreted briefly and analytical probabilistic models of raw transmissibility functions are developed. Section 4 presents some further remarks on the probabilistic models. In Section 5, the performance of the probabilistic models is verified by employing a numerical shear building model and two large-scale civil engineering structures under ambient vibration.

2. Revisiting theorems and corollaries

2.1. Theorem on multivariate circularly-symmetric complex normal ratio distribution

For the sake of simplicity, theorem 1 and theorem 2 in [42] can be combined and given as follows. Suppose that n_o- variate complex random vector $\mathbf{Y}\in\mathbb{C}^{n_o}$ with $\mathbf{Y}=\{Y_o,Y_1,\cdots,Y_{n_o-1}\}^T$ has multivariate circularly-symmetric complex normal distribution, then $\mathbf{U}\in\mathbb{C}^{n_o-1}$ with $\mathbf{U}=\{U_1,U_2,\cdots,U_{n_o-1}\}^T=\{Y_1/Y_o,Y_2/Y_o,\cdots,Y_{n_o-1}/Y_o\}^T$ follows a multivariate circularly-symmetric complex normal ratio distribution. The PDF of $\mathbf{U}=\mathbf{U}^{\mathfrak{R}}+\mathbf{i}\mathbf{U}^{\mathfrak{I}}\in\mathbb{C}^{n_o-1}$ and the real-valued vector $\Upsilon=\left[\left(\mathbf{U}^{\mathfrak{R}}\right)^T\ \left(\mathbf{U}^{\mathfrak{I}}\right)^T\right]^T\in\mathfrak{R}^{2(n_o-1)}$ are given by

$$p_{\mathbf{U}}(\mathbf{u})=\frac{(n_o-1)!}{\pi^{(n_o-1)}|\det(\mathbf{\Sigma})|\left(\tilde{\mathbf{u}}^*\mathbf{\Sigma}^{-1}\tilde{\mathbf{u}}\right)^{n_o}} \tag{1}$$

$$p_{\mathbf{U}}(\mathbf{u})=\frac{(n_o-1)!}{\pi^{(n_o-1)}\left|\tilde{\mathbf{\Sigma}}\right|^{1/2}\left[\boldsymbol{\eta}^T\tilde{\mathbf{\Sigma}}^{-1}\boldsymbol{\eta}\right]^{n_o}} \tag{2}$$

where $\tilde{\mathbf{u}}=[1,\mathbf{u}^T]^T=[1,u_1,u_2,\cdots,u_{n_o-1}]^T$. $\mathbf{\Sigma}$ is the covariance matrix of the complex random vector $\mathbf{Y}$; $\boldsymbol{\eta}=\left[\left(\tilde{\mathbf{u}}^{\mathfrak{R}}\right)^T\ \left(\tilde{\mathbf{u}}^{\mathfrak{I}}\right)^T\right]^T$ with $\tilde{\mathbf{u}}^{\mathfrak{R}}=\left\{1,(\mathbf{u}^{\mathfrak{R}})^T\right\}^T=\left\{1,u_1^{\mathfrak{R}},u_2^{\mathfrak{R}},\cdots,u_{n_o-1}^{\mathfrak{R}}\right\}^T$ and $\tilde{\mathbf{u}}^{\mathfrak{I}}=\left\{0,(\mathbf{u}^{\mathfrak{I}})^T\right\}^T=\left\{0,u_1^{\mathfrak{I}},u_2^{\mathfrak{I}},\cdots,u_{n_o-1}^{\mathfrak{I}}\right\}^T$. $\tilde{\mathbf{\Sigma}}=\frac{1}{2}\begin{bmatrix}\mathbf{\Sigma}^{\mathfrak{R}} & -\mathbf{\Sigma}^{\mathfrak{I}}\\ \mathbf{\Sigma}^{\mathfrak{I}} & \mathbf{\Sigma}^{\mathfrak{R}}\end{bmatrix}$ denotes the covariance matrix of real-valued vector $\left[\left(\mathbf{Y}^{\mathfrak{R}}\right)^T\ \left(\mathbf{Y}^{\mathfrak{I}}\right)^T\right]^T$.

2.2. Corollaries

For the special case of a bivariate random vector $\mathbf{Y}=\{Y_o,Y_1\}^T$ whose covariance matrix is given by $\mathbf{\Sigma}=\begin{bmatrix}\sigma_0^2 & \rho\sigma_0\sigma_1\\ \rho^*\sigma_0\sigma_1 & \sigma_1^2\end{bmatrix}$ with complex correlation coefficient $\rho=\rho^{\mathfrak{R}}+\mathbf{i}\rho^{\mathfrak{I}}=\beta e^{\mathbf{i}\alpha}$, one can prove that:

Corollary 1. The PDF of $U_1=Y_1/Y_o$ is equal to

$$p_{U_1}(u_1)\ =\pi^{-1}(1-\rho^*\rho)\sigma_0^2\sigma_1^2\left[\sigma_1^2-(u_1^*\rho^*+u_1\rho)\sigma_0\sigma_1+u_1u_1^*\sigma_0^2\right]^{-2} \tag{3}$$