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Robust fixed-order dynamic output feedback controller design for nonlinear uncertain suspension system

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ABSTRACT

This paper deals with designing a robust fixed-order non-fragile dynamic output feedback controller for active suspension system of a quarter-car, by means of convex optimization and linear matrix inequalities (LMIs). Our purpose is to design a low-order controller that keeps the desired design specifications besides the simplicity of the implementation. The proposed controller is capable of asymptotically stabilizing the closed-loop system and developing H_∞ control, despite model uncertainties and nonlinear dynamics of the quarter-car as well as the norm bounded perturbations of controller parameters. Furthermore, controller parameters are prevented from taking very large and undesirable amounts through appropriate LMI constraints. Finally, a numerical example is presented to show the effectiveness of the proposed method by comparing it with similar works.

1. Introduction

A vehicle's suspension system guarantees a smooth ride over rugged roads damping disturbances caused by the road surface, while ensuring the tyres remain in contact with the road surface and that body roll is minimized. Also, a suspension system allows the car to turn without rolling by changing the car's center of gravity to maintain balance. Therefore a comfortable and safe ride could be achieved [1].

Passive suspension systems which include conventional springs and dampers with fixed stiffness and damping coefficients are proved incapable of damping the road disturbances in all frequency ranges. Recently, controlled suspension systems, comprise of semi-active and active suspensions have been emerged due to unavoidable limitations of conventional suspensions. Semi-active suspension uses a controlled dissipative component rather than the damper and no energy input into the system is needed. While an active suspension needs to use an active actuator, and an adequate energy input. The advantages of active suspension systems compared with passive ones are presented in [2].

Design of automobile suspension system from the ride comfort point of view is reviewed in [3]. Subsequently a large number of studies have been performed to use the most appropriate control strategies acquiring different performance objectives.

In [4] two approaches are introduced in order to design centralized H_∞ controllers for an active magnetic suspension system. Semi-active H_∞ control of vehicle suspension is studied in [5]. In [6] the problem of robust sampled-data control for

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active suspensions considering input delay was investigated. Models utilized in [5,6] are considered to be linear and because of the non-linear nature of the springs used in the suspension system, this approximation can lead to inaccuracy. In [7] robust controller design with actuator saturation for a comprehensive nonlinear system model with parameter uncertainty in the presence of external disturbance was investigated. The proposed method therein, uses a nonlinear sign function for limiting the nonlinear active force. In [8] the concentration is also on sliding mode algorithms within the Lyapunov stability theorem, in which different practical control objectives are also considered. A robust fault-tolerant H_∞ controller with multiple control objectives for an active suspension system of a full-car model is designed in [9].

A strategy utilizing the proportional-integral sliding mode control scheme in controlling the active suspension system is presented in [10]. In [11] a control strategy based on model predictive control (MPC) methods for semi-active suspensions in road vehicles is designed. A fuzzy logic technique was used to control an active suspension system in [12], the ride comfort is also guaranteed by reducing the body acceleration. Practical feasibility of a new hybrid control method applied to an active suspension system of a quarter car model by skyhook and adaptive neuro active force control was investigated in [13]. An estimation of control voltage input to the magnetorheological damper is made in [14] by using neural network in order to reduce the vibrations.

A non-fragile static output feedback controller for active suspension systems, considering input delay, is investigated in [15], where the resulted constraints are in the form of bilinear matrix inequalities (BMIs), requiring iterative methods to be solved.

In [16] and [17] state feedback is used to design constrained H_∞ control for quarter-car active suspension system. However, using state feedback entails having all states which may sound difficult and costly in some cases.

Static output feedback controllers for a quarter-car active suspension model are designed in [18]. Also in [19] a robust non-fragile static output feedback controller is designed for a quarter-car with active suspension. However dynamic feedback is always preferable due to its more effective control performances.

A great deal of the controller design methods lead to high order controllers. Such systems have high implementation cost, poor reliability, high fragility, numerical error, and potential problems in maintenance. Plant or controller reduction techniques do not always guarantee that the closed-loop performance is preserved. Therefore, a challenging problem is to design directly a low-, fixed-order controller for a system [20]. Thus, by using a fixed order controller design method one can obtain simplicity and desired performance together.

There is no result on the LMI-based designing of a linear robust fixed-order dynamic output feedback controller with model uncertainty and nonlinear dynamics, for suspension systems of quarter-car models. In our proposed method we proved and showed that for the special case of active suspension system, with a full linear controller, desired specifications are obtainable through LMI optimization even with considering a time-varying delay in the control input. Besides, the approach for saturating the active force is also within an LMI viewpoint and thus remains linear. Furthermore, to the best of our knowledge considering the most complete model of linear controller with direct feedthrough parameter, is not available on earlier similar researches. It is worth noting that norm-bounded perturbations of controller parameters can also be tolerated using the proposed method.

The organization of the remainder of this paper is as follows. Problem statement as well as primary backgrounds such as introducing the model and performance objectives is provided in Section 2. Section 3 is devoted to the main results of the paper including problem formulation and controller design in the form of a theorem and a corollary. Some numerical examples are provided in Section 4 to demonstrate the effectiveness of the proposed method. Eventually, conclusions are discussed in Section 5.

2. Problem statement and preliminaries

A vehicle can be modeled as two sub-systems including sprung mass and unsprung mass joined together with some elastic and dissipative elements such as suspensions and tyres. Sprung mass refers to chassis mass imposed to springs and dampers, while unsprung mass is the mass of wheels, axles, and connections placed between springs and road surface. Aforesaid two degree-of-freedom model is influenced by disturbances and inputs caused by road surface.

Suspension dynamics of an active quarter-vehicle can be modeled with the relationship of mass spring damper components as depicted in Fig. 1. Moreover, required parameters of a quarter car suspension system are provided in Table 1. In this model the wheel is represented by the tyre, with spring and damping characteristics.

The tyre also plays a low-pass filter role in the suspension system reducing the high-frequency shakes caused by road surface [21]. Assuming no wheel lift-off and no slippage, one can obtain the dynamic equations of vehicle suspension system [22].

$$\begin{aligned} &m_s\ddot{x}_s(t) + c_s[\dot{x}_s(t) - \dot{x}_u(t)] + k_s[x_s(t) - x_u(t)] + k_{ns}[x_s(t) - x_u(t)]^3 = u(t - \tau(t)) \\ &m_u\ddot{x}_u(t) + c_s[\dot{x}_u(t) - \dot{x}_s(t)] + k_s[x_u(t) - x_s(t)] + k_{ns}[x_u(t) - x_s(t)]^3 + k_t[x_u(t) - x_r(t)] \\ &+ c_t[\dot{x}_u(t) - \dot{x}_r(t)] = -u(t - \tau(t)) \end{aligned} \tag{1}$$

In which $\tau(t)$ is a time-varying delay by the condition of $\dot{\tau}(t) < \bar{\tau}_d < 1$.

Undoubtedly the principal task of the suspension system is to minimize the vertical acceleration sensed by the rider which brings about ride comfort and less depreciation. Therefore the body acceleration $\ddot{x}_S(t)$ is taken as the first control