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DOA estimation of closely-spaced and spectrally-overlapped sources using a STFT-based MUSIC algorithm ☆

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ABSTRACT

The multiple signal classification (MUSIC) algorithm based on spatial time-frequency distribution (STFD) has been investigated for direction of arrival (DOA) estimation of closely-spaced sources. However, the limitations of the bilinear time-frequency based MUSIC (TF-MUSIC) algorithm lie in that it suffers from heavy implementation complexity, and its performance strongly depends on appropriate selection of auto-term location of the sources in time-frequency (TF) domain for the formulation of a group of STFD matrices, which is practically difficult especially when the sources are spectrally-overlapped. In order to relax these limitations, this paper aims to develop a novel DOA estimation algorithm. Specifically, we build a MUSIC algorithm based on spatial short-time Fourier transform (STFT), which effectively reduces implementation cost. More importantly, we propose an efficient method to precisely select single-source auto-term location for constructing the STFD matrices of each source. In addition to low complexity, the main advantage of the proposed STFT-MUSIC algorithm compared to some existing ones is that it can better deal with closely-spaced sources whose spectral contents are highly overlapped in TF domain.

1. Introduction

The direction of arrival (DOA) estimation has received tremendous research attention in different areas, such as radar, sonar, speech, and communication [1–6]. The multiple signal classification (MUSIC) algorithm and its variants [7–18] based on the eigen-structure of the observed covariance matrix have been efficient estimation techniques. Nevertheless, the MUSIC algorithm has limitations with regard to resolving closely-spaced sources in low SNR environments. In addition, it can only work when the number of sensors is larger than the number of sources. With the development of spatial time-frequency distributions (STFDs) [19–24], the conventional MUSIC based on space-time processing has been developed by constructing STFD matrices instead of the covariance matrices for narrowband DOA estimation [25–32]. It has also been extended for wideband scenarios [33]. Compared to the temporal MUSIC, the bilinear time-frequency based MUSIC (TF-MUSIC) can provide noise-robust direction finding since the noise power is spread over the time-frequency (TF) domain while the source energy is localized [34]. Moreover, the selection of TF region belonging to a specific set of sources allows the improved DOA estimation with a small number of sensors in an underdetermined case. By separately dealing with the auto-source TF points of each source, the DOAs of closely-spaced sources can be estimated as long as they are distinctly distributed in TF domain.

However, the TF-MUSIC[1] is computationally complicated compared to the temporal MUSIC, and it is highly dependent on the selection of auto-term location of the sources in TF domain[2] to

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[1] Herein the TF-MUSIC means the MUSIC algorithm based on pseudo Wigner–Ville distribution (PWVD) unless otherwise specified.

[2] We term the auto-term location as auto-source TF points in this paper.

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mitigate the noise effect and ensure the full rank property of the STFD matrix. Multiple auto-source TF points [25,27] or cross-source TF points [26] have been considered to construct the STFD matrix. Most of previous studies assume that the source signals are sparse in TF domain and the TF signature of each source is known. However, the precise information of auto-source TF points in practical situations might be unavailable, and the estimation of the TF signature of individual source is required. This problem becomes significant for scenarios involving a large number of sources, whose spectral contents are overlapped in TF domain. Therefore, an estimation of appropriate auto-source TF points becomes practically crucial. Some efficient selection methods have been reported. In [35], Heidenreich et al. applied morphological image processing to detect instantaneous frequency (IF) segments of each source. The detected IF segments are combined based on a bootstrap resampling technique, and the linking IF segments belonging to a single source are then used for DOA estimation. Other TF point selection methods based on quadratic STFDs can refer to [29,36,37]. Due to the computational complexity and the cross-terms of quadratic time-frequency distributions (TFDs), the linear TFD-based DOA estimation techniques, e.g., the fractional Fourier transform (FRFT) in [38] and the short-time Fourier transform (STFT) in [39], have also been investigated in the literature. Some methods of selecting signal-source TF points based on linear TFDs of mixtures have been reported in [40–44]. However, they have some limitations, i.e., the method in [40,41] is only suitable for two mixtures, and requires the sources to be W-disjoint orthogonal in TF domain. The methods in [42–44] are designed in the case of real-valued mixing matrix. More sophisticated selection methods in the presence of complex-valued mixing matrix are strongly required particularly for the case where the sources are spectrally-overlapped in a low SNR environment.

To ease the limitations of the temporal MUSIC and the bilinear TF-MUSIC, herein we propose a short-time Fourier transform based MUSIC (STFT-MUSIC) algorithm, aiming at solving the challenging scenarios associated with a large number of closely-spaced and spectrally-overlapped sources. The research emphasis is on the STFT because it is simple to implement and the cross-term is avoided. The main contribution in the paper is that an efficient method to automatically select single-source TF points (i.e., the TF points where only one source exists) is proposed. Instead of detecting the TF points along the IF trajectory, we propose a subspace projection based method to detect single-source TF points of each source in the presence of complex-valued mixing matrix. Although the linear STFT suffers low resolution in TF domain, appropriate single-source TF points can be selected for accurate DOA estimation since the proposed algorithm is not focused on the signal's IF laws. Compared to the temporal MUSIC and the bilinear TF-MUSIC, the advantages of the proposed STFT-MUSIC algorithm are four-fold:

- The ability to resolve the closely-spaced sources, which is the limitation of the temporal MUSIC.
- The feasibility in underdetermined cases, which is the limitation of the temporal MUSIC.
- The feasibility and efficiency to deal with spectrally-overlapped sources, which are the limitations of the TF-MUSIC.
- The low complexity, which is the limitation of the TF-MUSIC.

The remainder of this paper is structured as follows. We describe the STFT-MUSIC algorithm in Section 2. In Section 3, the details of the proposed method of how to choose single-source TF points are given. In Section 4, the STFT-MUSIC algorithm with selected single-source TF points is evaluated on two scenarios, and the comparison with existing algorithms is presented. The discussion of the proposed algorithm with existing work is given in Section 5. Finally, Section 6 concludes the paper.

2. The proposed STFT-MUSIC algorithm

In our study, a linear and equispaced sensor array having M elements is considered, i.e., the spatial distribution of sensors is uniform linear array (ULA). Let $s_i(t), i = 1, \ldots, N$, denote the unknown sources, where N is the number of sources impinging on an M-dimensional ULA from N distinct directions $\theta_1, \ldots, \theta_N$. The output signals $x_m(t), m = 1, \ldots, M$ are modeled as

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{A}(\theta) = \left[\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_N)\right]$ denotes the complex-valued mixing matrix, and $\mathbf{a}(\theta_i)$ is the steering vector of the ith source. $\mathbf{x}(t) = \left[x_1(t), \ldots, x_M(t)\right]^T$ are the received mixtures, $\mathbf{s}(t) = \left[s_1(t), \ldots, s_N(t)\right]^T$ are the signal sources, and $\mathbf{n}(t)$ is the additive white Gaussian noise vector. $[\cdot]^T$ is the transpose operator.

The STFT (denoted by $\mathbf{S}$ hereafter) of the array output vector $\mathbf{x}(t)$ in (1) without noise is computed as

$$\mathbf{S_x}(t, f) = \mathbf{A}(\theta)\mathbf{S_s}(t, f) = \left[\mathbf{a}(\theta_1) \quad \ldots \quad \mathbf{a}(\theta_N)\right] \begin{bmatrix} \mathbf{S}_{s_1}(t, f) \\ \vdots \\ \mathbf{S}_{s_N}(t, f) \end{bmatrix}, \tag{2}$$

where $\mathbf{S}_{s_i}(t, f)$ denotes the STFT value of the ith source. The steering vector of the ith source denotes $\mathbf{a}(\theta_i) = [a_{i1}, \ldots, a_{iM}]^T$, and its mth element is expressed as

$$a_{im} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{\lambda}d(m-1)sin(\theta_i)}, \quad m \in \{1, \ldots, M\}, \tag{3}$$

where d is the inter-element spacing, λ denotes the wavelength, and θ_i is the DOA of the ith source to be estimated.

Define $\mathbf{D_{xx}}(t, f)$ to be formulated as

$$\begin{aligned} \mathbf{D_{xx}}(t, f) &= \mathbf{S_x}(t, f)\mathbf{S_x}^H(t, f) = \mathbf{A}(\theta)\mathbf{D_{ss}}(t, f)\mathbf{A}^H(\theta) \\ &= \mathbf{A}(\theta) \begin{bmatrix} \mathbf{S}_{s_1}(t, f)\mathbf{S}_{s_1}^*(t, f) & \ldots & \mathbf{S}_{s_1}(t, f)\mathbf{S}_{s_N}^*(t, f) \\ \vdots & \vdots & \vdots \\ \mathbf{S}_{s_N}(t, f)\mathbf{S}_{s_1}^*(t, f) & \ldots & \mathbf{S}_{s_N}(t, f)\mathbf{S}_{s_N}^*(t, f) \end{bmatrix} \\ &\quad \times \mathbf{A}^H(\theta), \end{aligned} \tag{4}$$

where $\mathbf{D_{ss}}(t, f)$ denotes the source STFD matrix whose (i, j)th element is $\mathbf{D}_{s_i s_j}(t, f) = \mathbf{S}_{s_i}(t, f)\mathbf{S}_{s_j}^*(t, f)$. The conjugate operator is denoted by $[\cdot]^*$.

The STFT-MUSIC algorithm estimates the DOA by determining the N peaks of the spatial spectrum

$$P(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta)}, \tag{5}$$

where $\mathbf{U}_n$ denotes the noise eigenvectors of the STFD matrices in (4), which are replaced by the covariance matrices of $\mathbf{x}(t)$ in the conventional MUSIC algorithm [9]. The conjugate transpose operator is denoted by $[\cdot]^H$. The STFD matrices lead to an enhanced SNR, and therefore to an improved accuracy of DOA estimation [27]. Assuming a group of single-source TF point sets $\Omega_i, i = 1, \ldots, N$, for individual sources are determined, the averaged STFD matrices are obtained by

$$\overline{\mathbf{D}}_i = \frac{1}{\sharp\Omega_i} \sum_{(t,f)\in\Omega_i} \mathbf{D_{xx}}(t, f), \quad i = 1, \ldots, N, \tag{6}$$

where $\sharp\Omega_i$ denotes the number of TF points in the set Ω_i, and $\mathbf{D_{xx}}(t, f)$ is the STFD matrix at the TF point (t, f). In the STFT-MUSIC algorithm, the noise subspace $\mathbf{U}_n$ in (5) is therefore computed by the eigen-decomposition of the matrix $\overline{\mathbf{D}}_i$.