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Adaptive digital ridgelet transform and its application in image denoising

Qiangui Huang, Boya Hao, Sheng Chang *

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ABSTRACT

In this paper, we propose a new multiscale decomposition algorithm called adaptive digital ridgelet (ADR) transform. Differently from the traditional nonadaptive multiscale decompositions, this algorithm can adaptively deal with line and curve information in an image by considering its underlying structure. As the key part of the adaptive analysis, the curve parts of an image are detected accurately by a new curve part detection method. ADR transform is applied to image denoising experiment in this paper. Experimental results demonstrate its efficiency for reducing noises as PSNR values can be improved maximally 5 dB compared with other methods and MAE values are also considerably improved. A new comparison criterion is also proposed and using this criterion, it is shown that ADR transform can provide a better performance in image denoising.

1. Introduction

With the rapid development of image processing, more and more tasks, such as image restoration, edge detection, singularity detection, image compression, texture recognition and classification, exceed the ability of traditional Fourier transform as an image analysis method. Its limitations in approximation accuracy and directional information description remain this challenge unsolved until the flourishing of multiscale decomposition analysis in recent years.

In the past decades, the development of multiscale decomposition analysis has built many new tools for image analysis, such as wavelet [1], ridgelet [2–4], curvelet [5,6], brushlet [7], wedgelet [8], beamlet [9], contourlet [10], bandelet [11], directionlet [12], and shearlet [13] methods. Different prominent properties of these methods make singularity analysis become possible and benefit image processing a lot. Among the family of the multiscale geometric transforms, ridgelet and curvelet transforms have been the most widely used ones due to their ideal approximation property in linear and curved singularity analysis.

In [6], Candes utilized two terminologies, Lagrangian representation and Eulerian representation, to elaborate the distinction of multiscale transform. The former one denotes the representation is constructed using full knowledge of the intrinsic structure of an object and the latter one indicates a fixed nonadaptive construction of a given object. Although Lagrangian representation yields a better approximation performance, normally it is hard to get a thorough knowledge of an image. So actually, all the multiscale geometric analysis methods are Eulerian representations.

Candes and Donoho stated in [2,5,6] that ridgelet is the best approximation method to represent an object which has a discontinuity across a line, and curvelet is the best one for objects with discontinuities across curves. The approximation rates of ridgelet and curvelet are very close to the ideal Lagrangian condition, and are better than any other nonadaptive approaches such as Fourier transform and wavelet transform. Because of that, ridgelet transform and curvelet transform are widely used in image analysis, such as image denoising and enhancement [14–17], texture classification [18,19], and watermarking [20].

However, ridgelet transform and curvelet transform are nonadaptive methods and majority of their applications apply nonadaptive processing to research objects too. This fact inherently limits their analysis ability. Behind the prosperous applications in various images, a few deep thoughts about how to improve their performance are presented.

To overcome this dilemma, a new algorithm, adaptive digital ridgelet (ADR) transform, is proposed in this paper. It first takes the underlying structure of an image into consideration and then deals with line and curve information adaptively. This new decomposition strategy provides us another solution to analyze images adaptively. And various experimental results demonstrated it is very efficient in image denoising.

The rest of this paper is organized as follows. In Section 2, we expound the adaptive digital ridgelet transform algorithm. Image

* Correspondence to: Department of Electronics Science and Technology, School of Physics and Technology, Wuhan University, Wuhan, Hubei 430072, China.
E-mail address: changsheng@whu.edu.cn (S. Chang).

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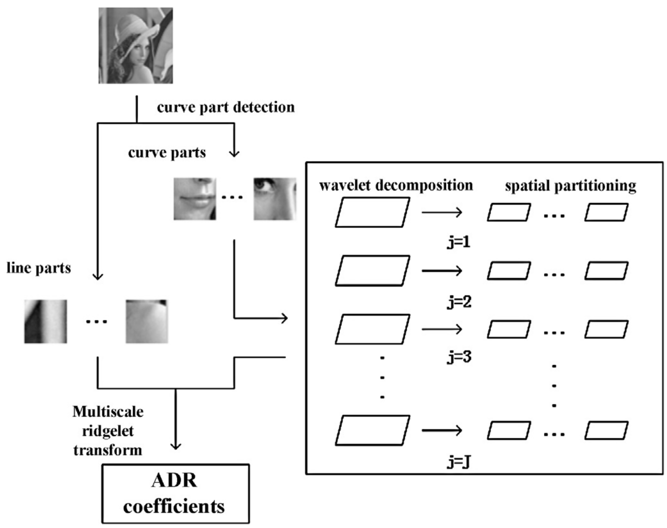

Fig. 1. Frame diagram of ADR transform.

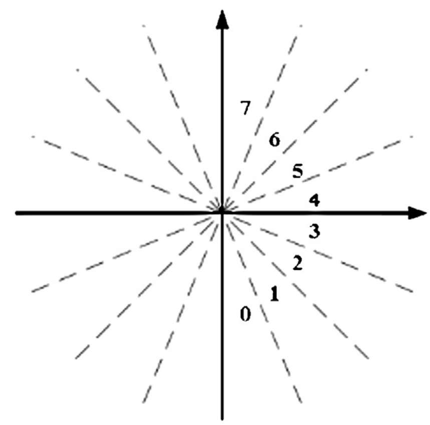

Fig. 2. Sketch map of coordinate partition for orientation field values.

denoising methodology for ADR transform is presented in Section 3, and experiment results are reported and discussed in Section 4. Finally, we draw conclusions in Section 5.

2. Adaptive digital ridgelet transform

In this section, concept and detailed processing procedure of ADR transform is illustrated. Although it is impossible for us to get totally full knowledge of intrinsic structure of an object, it could be much easier for one to concentrate on some interested parts of an image only such as line and curve information.

Usually, line and curve information are both rich and crucial in our research targets. It is not hard to imagine that if analysis can be adaptive to them and process them separately, researches can be more effective. Motivated by this, an adaptive digital ridgelet transform is proposed, which can deal with line and curve information adaptively. A frame diagram of ADR transform is presented in Fig. 1.

As shown in the frame diagram, with the assistance of a curve part detection algorithm, the original image is separated into a series of line and curve parts, in which line or curve is the major component respectively. Digital ridgelet transform can be used in line parts directly due to its excellent property in linear singularity analysis. However, it is not suitable for curve parts. Inspired by the methodology of curvelet, wavelet transform is applied to decompose the curve parts into a finer scale. Here, $j = J$ indicated the finest scale. Next the coefficients are spatially partitioned into squares in different scales. After these, the curved singularity has been converted to linear singularity. Consequently, ridgelet transform is utilized for the original curve parts too.

In the method above, one can think that using curvelets seems to be more appropriate than transforming curved singularities into the linear singularities. However, directly applying curvelet transform to curve parts will result in serious block effect after restoration. All partitioned parts are non-overlapped. Once reconstructing them after finishing the coefficient processing procedure, there could be block effects because line and curve parts are decomposed by two different transforms. To mitigate the block effect, here we apply digital ridgelet transform to curve parts too. The following part is the detailed subsection about curve part detection and how to implement the decomposition.

2.1. *Curve part detection*

An orientation field represents an intrinsic property of image's underlying structure and defines the gray intensity change in a local neighborhood. In image analysis, orientation fields have been widely used. After the observation of different types of images and their orientation fields, an empirical phenomenon is discovered that curves in an image usually trigger some changes in orientation field. The reason is not hard to imagine. Generally, the curvature value of a curve is obviously larger than the one of a line, while the curvature is exactly zero at every point in a straight line. Hence, for a curve, there will be some changes in curvature which can be reflected by the difference of orientation values. And that's where a curve "turns around".

Based on this, the curve turning-around points can be detected by searching the orientation corner points where the orientation fields change. And this kind of change can be indicated by the difference of orientation fields. Afterwards, parts in an image with rich curve information can be localized by these curve turning-around points.

In different applications of image analysis, a variety of orientation field computational methods have been developed. In this paper, a relatively easy and effective one is utilized which is proposed by Lin in fingerprint analysis [21]. For a given image F, the block orientation O can be obtained by Lin's algorithm where each value represents the orientation of its corresponding $w \times w$ block and lies in $[-\pi/2, \pi/2]$. For further processing procedure, orientation values in O are converted into $[0, 7]$, where original values in $[-\pi/2, -3\pi/8]$, $[-3\pi/8, 2\pi/8], \dots, [3\pi/8, \pi/2]$, are altered into $0, 1, \dots, 7$, respectively. This means the coordinates are evenly separated into 8 parts and 0–7 represents one corresponding part as shown in Fig. 2. An example of orientation field computed by methods in [21] is presented in Fig. 3.

Consequently, in order to detect the curve part of F, F is partitioned into a series of $L \times L$ blocks without overlap. The detection result of each block is computed by formula (1).

$$label(i, j) = op\big(d\_h(O) + d\_v(O) + d\_d(O)\big)$$
$$O = \{O_{i,j,1,1}, \dots, O_{i,j,L/w,L/w}\} \quad i, j = 1, 2, \dots, L \tag{1}$$

Here, $label(i, j)$ represents the detection result of the block in ith row and jth column. If its value is one, it means this is a curve block. Otherwise, it is detected as a line block. $\{O_{i,j,1,1}, \dots, O_{i,j,L/w,L/w}\}$ indicates the $L/w \times L/w$ orientation values in one partitioned block. d_h, d_v, and d_d denote the differential operation in horizontal, vertical, and diagonal direction, respectively, which could be calculated by simply subtracting current orientation value and previous one in horizontal, vertical,