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# Time-synchronous-averaging of gear-meshing-vibration transducer responses for elimination of harmonic contributions from the mating gear and the gear pair

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ABSTRACT

The transmission-error frequency spectrum of meshing gear pairs, operating at constant speed and constant loading, is decomposed into harmonics arising from the fundamental period of the gear pair, rotational harmonics of the individual gears of the pair, and tooth-meshing harmonics. In the case of hunting-tooth gear pairs, no rotational harmonics from the individual gears, other than the tooth-meshing harmonics, are shown to occur at the same frequencies. Time-synchronous averages utilizing a number of *contiguous* revolutions of the gear of interest equal to an integer multiple of the number of teeth on the mating gear is shown to eliminate non-tooth-meshing transmission-error rotational-harmonic contributions from the mating gear, and those from the gear pair, in the case of hunting-tooth gear pairs, and to minimize these contributions in the case of non-hunting-tooth gear pairs. An example computation is shown to illustrate the effectiveness of the suggested time-synchronous-averaging procedure.



## 1. Introduction

The static transmission error is widely accepted to be the principal source of vibration generated by meshing gear pairs [1–9]. Any defect (damage) on a loaded tooth-working-surface of a meshing gear will yield a contribution to the transmission error. To detect such damage, gear-health monitoring is carried out by processing the vibration responses of transducers hard-mounted on gear supporting structures.

If the earliest possible detection of damage on a particular gear is desired, it is necessary to isolate the transducer-vibration-response of that gear from the response of the gear mating to the gear of interest, and from other meshing gears in the system. This isolation is carried out by forming a *coherent average* of the transducer response to many revolutions of the gear of interest, by forming a *time-synchronous average* of these revolutions [10–23]. In particular, McFadden [15] has described this procedure for general periodic signals corrupted by noise, by utilizing a comb-filter model of time-domain averaging, applying a rectangular window to the noisy signal. By selection of an appropriate number of averages, he has shown that periodic noise of a known frequency can be eliminated. In contrast, the analysis described below is specifically applied to meshing gear pairs, thereby identifying several sets of corrupting harmonics that are shown to be eliminated by appropriate choice of the number of contiguous segments to be utilized in the time-synchronous averages. By utilizing an

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increasing multiple of the number of revolutions of the gear of interest in such averages, more of the interference from other gears in the system is averaged out.

In order for potential wear on meshing gear teeth to be uniformly distributed, most power-transmission gear pairs have hunting teeth – i.e., the number of teeth on each of the meshing gears can have no common integer divisor except unity. A simple proof is provided to show that the only rotational-harmonic frequency contributions from each gear of a meshing hunting-tooth pair, common to both gears of the pair, are the tooth-meshing-harmonic contributions of the gear pair [18,19].

Because real gears have tooth-spacing and other errors, and small misalignments between the (base cylinder) axis used in manufacturing and the bearing axis used in operation, the fundamental transmission-error period of a meshing *gear pair* is the interval required for particular teeth on each of the two gears to again come into contact at the same contact points [4]. Numerous additional weak transmission-error harmonic contributions are shown to be associated with this fundamental period of the gear pair.

By utilizing the number of *contiguous* revolutions of the gear of interest, in the synchronous average, equal to an integer multiple of the number of teeth on the mating gear, it is shown that all of these weak harmonics of the gear pair, and those of the mating gear to the gear of interest, are eliminated in such a time-synchronous average in the case of hunting-tooth gear pairs, leaving only the transmission-error rotational-harmonic contributions of the gear of interest and the tooth-meshing-harmonic contributions of the gear pair. For non-hunting-tooth gear pairs, some rotational-harmonic contributions from the mating gear to the gear of interest remain in the time-synchronous average, and occupy the same frequency locations as those of the gear of interest.

The results derived herein, showing the specific number of contiguous segments to be utilized in the time-synchronous averages required for elimination of rotational-harmonic contributions from the mating gear and the gear pair, have been utilized in earlier works [24,29], but without the proof provided herein. A result from [24] illustrating the effectiveness of this synchronous-averaging procedure is provided.

## 2. Transmission-error contributions from damaged teeth

According to Gregory et al. [2], “the transmission error is defined, for any instantaneous angular position of one gear, as the angular displacement of the mating gear from the position it would occupy if the teeth were rigid and unmodified.” For the present application to gear-health monitoring, an alternative definition of transmission error applicable to parallel-axis involute gears, essentially equivalent to that given above, is convenient. Fig. 1 illustrates an idealized pair of meshing parallel-axis helical or spur gears with rigid unmodified equispaced perfect involute teeth, with base circle radii $R_b^{(1)}$ and $R_b^{(2)}$. Such idealized involute gears would transmit an *exactly* constant speed ratio. Let $\theta^{(1)}$ and $\theta^{(2)}$ denote the instantaneous angular positions of the two *perfect* gears, and $\delta\theta^{(1)}$ and $\delta\theta^{(2)}$ the instantaneous deviations of the rotational positions of the two gears from the positions of their rigid perfect involute counterparts, where these angular positions are “measured” in radians. Assume positions of the gear shafts remain stationary and perfectly parallel. A single variable

$$x \triangleq R_b^{(1)}\theta^{(1)} = R_b^{(2)}\theta^{(2)}, \tag{1}$$

describes the instantaneous rotational positions of the two perfect involute gears. Following [1], define the transmission error as positive when mating teeth “come together” relative to their rigid perfect involute counterparts as arises, for example, in tooth elastic deformations under the loading $W$. Then the lineal transmission error $\zeta(x)$ can be defined [25,9] as

$$\zeta(x) \triangleq R_b^{(1)}\delta\theta^{(1)}(x) - R_b^{(2)}\delta\theta^{(2)}(x), \tag{2}$$

where the negative sign arises from the sign convention of $\theta^{(2)}$ in Fig. 1, and where $\zeta(x)$ will vary with the instantaneous rotational positions of the two gears, designated by $x$. Eq. (2) describes the lineal distance that the two gears approach one another in the plane of contact in Fig. 1 relative to their rigid perfect involute counterparts.

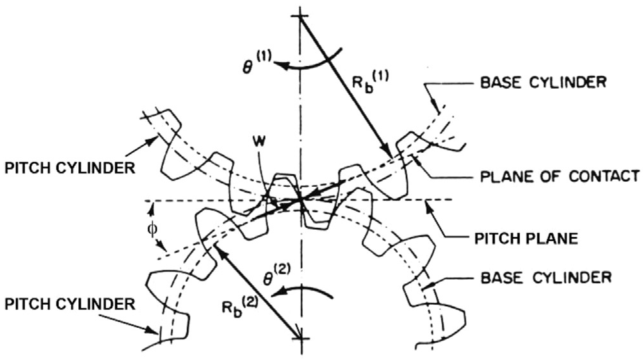


**Fig. 1.** Pair of parallel-axis perfect involute gears.