Mechanical Systems and Signal Processing 62-63 (2015) 183–206

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# Time-frequency characterization of lamb waves for material evaluation and damage inspection of plates

P. Frank Pai [a,*], Haoguang Deng [a], Mannur J. Sundaresan [b]

[a] Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA
[b] Department of Mechanical Engineering, North Carolina A&T State University, Greensboro, NC 27411, USA

ARTICLE INFO

*Article history:*
Received 28 March 2014
Received in revised form
3 February 2015
Accepted 10 March 2015
Available online 4 April 2015

*Keywords:*
Lamb wave dynamics
Time-frequency analysis
Guided wave-based damage inspection
Material characterization

ABSTRACT

Guided wave-based technique is one major approach for damage inspection of structures. To detect a small damage, an elastic wave's wavelength needs to be in the order of the damage size and hence the frequency needs to be high. Unfortunately, high-frequency wave dynamics always involves complicated wave reflection, refraction and diffraction, and it is difficult to separate them in order to perform detailed examination and system identification. This paper investigates dynamic characteristics of Lamb waves in plates in order to be used for material evaluation and damage inspection of thin-walled structures. A one-dimensional finite-element modeling and analysis technique is developed for computing dispersion curves and all symmetric and antisymmetric modes of Lamb waves in isotropic and multi-layer plates. Moreover, the conjugate-pair decomposition (CPD) method is introduced for time-frequency analysis of propagating Lamb waves. Results show that, under a *k*-cycle sine-burst excitation at a plate's edge, the time-varying frequency of a surface point's response can reveal the Lamb wave propagating inside the plate being a symmetric or an antisymmetric mode. The frequency of the measured wave packet increases from the wave front to the trailing edge if it is a symmetric mode, and the frequency decreases from the wave front to the trailing edge if it is an antisymmetric mode. Moreover, interaction of two different wave packets results in a peak in the time-frequency curve. These characteristics can be used for accurate separation of wave packets and identification of different wave speeds to enable fast and accurate material evaluation and damage inspection. Transient finite-element analysis of Lamb waves in finite plates with crack/delamination show that *k*-cycle sine-burst probing waves are good agents for guided wave-based damage inspection of structures. Although crack and delamination introduce different waves into and complicate the probing wave packet, time-frequency analysis makes it possible to separate such damage-induced small waves from the probing wave and enable fast and accurate damage inspection of thin-walled structures.



## 1. Introduction

For an elastic body, two or more parallel boundaries introduce one or more characteristic dimensions into its wave propagation mechanics, make it a waveguide, and cause the phase speed of a harmonic wave propagating in it to be

* Corresponding author.

http://dx.doi.org/10.1016/j.ymssp.2015.03.011
0888-3270/

frequency-dependent and a wave packet consisting of multiple harmonics to be dispersive. Hence, elastic waves become good agents for material characterization and damage inspection of thin-walled structures. Different from *bulk waves*, *guided waves* interact with structural boundaries/damages through reflection, refraction and diffraction, and mode conversion may occur between P-waves (primary, longitudinal waves) and S-waves (secondary, shear waves). A finite body can support an infinite number of different wave modes. *Lamb waves* are formed by interference of multiple reflections and mode conversion between P- and S-waves in a plate without traction forces on its upper and lower surfaces, and they are dispersive [1,2]. *Rayleigh waves* exist around the free surface of a semi-infinite solid, and they decay with the depth. *Stonely waves* occur at the interface of two different medias, where continuity of stresses and displacements is required at the interface and a radiation condition must be satisfied [1]. By matching theoretical dispersion curves of Lamb waves with experimental ones, material properties (e.g., Young's modulus, Poisson's ratio and mass density) can be determined [1–4]. Hence, Lamb waves have been used in ultrasonic NDT applications including material characterization of elastic and viscoelastic plates [3,5], bonding inspection [6], coating inspection [7], defect inspection [8], and thickness measurement of thin films [9]. However, almost all these methods are based on comparing measured dispersion curves and/or global wave speeds with theoretical ones to roughly deduce system parameters without detailed time-frequency analysis of the propagating wave profiles.

Many dynamics-based structural health monitoring (SHM) methods have been developed for different levels of damage inspection using different frequency ranges and dynamic properties [10–17]. Sensitivity to damage generally increases with frequency because direct detection of a defect without too much post-processing requires the associated wavelength being in the order of the defect's characteristic length. In the low-frequency range, there are vibration-based methods using conventional vibration testing techniques for damage inspection [15–24]. These methods are often not sensitive to small defects because of long wavelengths, and hence detail and intensive post-measurement signal processing is needed for extracting damage indicators and/or system parameters [23,24]. In the medium-frequency range, electro-mechanical impedance (EMI) methods use the mid-range frequencies between 10 kHz and 500 kHz for excitation and response analysis [25–28]. EMI methods often use a PZT patch for actuation and sensing. The electrical impedance (a complex voltage/current ratio as a function of frequency) of the PZT is directly affected by the local mechanical impedance (a complex force/velocity ratio) of the host structure, and the host structure's impedance is changed when damage exists. Existence of damage is revealed by comparing the PZT's impedance of the undamaged PZT-structure system with the one of the damaged system. However, the frequency range that reveals damage is case-dependent and is typically selected by trial and error, but the one contains 20–30 peaks in the real impedance is often chosen to ensure sufficient structural information [26,27]. An impedance method is easy to use because it requires not much equipment and results can be acquired without much post-processing of response signals. Unfortunately, because the measured impedance changes are unknown complex functions of physical parameters, it is difficult to deduce damage degree and locations (especially those away from the PZT) [26]. In the high-frequency range, there are guided wave (GW) methods based on propagation dynamics of a probing elastic wave of a short duration in the megahertz range. These methods are generally more sensitive to small damages because of short wavelengths, but their response signals are often difficult to analyze because of wave reflection, refraction and diffraction and their couplings, especially in large and/or complex structures [1]. Hence, such GW-based methods for system identification and/or damage inspection often depend on the use of the initial propagation response before reflection and refraction complicate the probing wave. Because such initial response is essentially transient and has a very short time duration and damage introduces extra waves that may make the transient period even shorter, time-frequency analysis is almost the only way for detailed dynamics characterization. On the other hand, vibration-based methods never intend to and are incapable of extracting structural information from the beginning wave dynamics right after the application of excitation.

Today's signal processing for system identification and damage inspection is leading toward time-frequency analysis with the main goal to enable processing of transient response and to increase the time-domain resolution without sacrificing the frequency-domain resolution. Time-frequency analysis is needed for linear time-varying (LTV) systems and nonlinear systems that are often used in engineering [29]. Parametric identification of LTV systems has been widely studied in the last two decades [30–32], but most of them are based on sectional linearization, which is inaccurate and computationally awkward. On the other hand, Feldman [18,33] proposed a method to identify time-varying nonlinear systems using Hilbert transform. Shi et al. [22] utilized the empirical mode decomposition (EMD) of Hilbert-Huang transform (HHT) [20] to identify time-varying system parameters by using free and forced responses. Unfortunately, EMD is incapable of decomposing a signal having multiple components of close frequencies and the accuracy of HHT for time-frequency analysis seriously suffers from the edge effect [24].

Due to its capability of multi-resolution analysis, wavelet transform has been widely studied for linear and non-linear system identification [15–17,19,34–36]. According to the Heisenberg uncertainty principle, unfortunately, wavelet transform is incapable of simultaneously providing high resolutions in both time and frequency domains [37]. Hence, it is difficult to use wavelet-based methods for dynamics characterization and system identification of a dynamical system having close and/or fast varying modal frequencies. Another method for time-frequency analysis is Wigner–Ville distribution (WVD), which is a quadratic time-frequency representation of a dynamical signal and its time-frequency resolution is not constrained by the Heisenberg uncertainty principle [38]. Unfortunately, the accuracy of WVD seriously suffers from the cross terms when the processed signal contains multiple components even though methods have been proposed to reduce the influences of cross terms [39–41].