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An experimental assessment of internal variables constitutive models for viscoelastic materials

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ABSTRACT

The present work is aimed at presenting an experimental assessment of a constitutive model used to describe viscoelastic behavior. This strategy is built on the basic principles of the Verification and Validation (V&V) philosophy. The mechanical model used to describe the viscoelastic behavior is a constitutive one based on the concept of internal variables. The parameter estimation of the model is performed using frequency domain data through the Particle Swarm Optimization algorithm. A set of different experimental set-ups were built in order to span the structural operational domain from which data can be measured. The model validation is performed based on the use of validation metrics which take into account uncertainties both in the model predictions and observed data.

1. Introduction

Computational models (CM) have been broadly used in different areas of engineering, applied sciences, industry and academia. They have been employed for preliminary design, optimization and decision-making. Such increasing reliance on computer model predictions has led to the nucleation and development of the Verification and Validation (V&V) field [1,2].

A non exhaustive literature review is presented in the sequel featuring recent articles based on the basic principles of V&V applied to different areas. Liang et al. [3] present some methodologies to calibrate power loads and validate power distribution system models. Greenwald [4] presents a comprehensive text containing some information about V&V, emphasizing the usefulness of V&V for physics of plasmas inasmuch as this area encompasses different ranges of temporal and spatial scales, nonlinearities and extreme anisotropy. Hemez et al. [5] propose and define some desired characteristics for what is called predictive maturity and present metrics to track model progress as additional information becomes available. Jiang et al. [6] present a Bayesian nonlinear structural equation modeling approach to hierarchical assessment of mechanical systems. Jiang and Mahadevan [7] propose a wavelet spectral analysis based validation approach. Castello et al. [8] propose an approach to estimate the constitutive parameters of a viscoelastic model based on internal variables in which validation analysis was performed based on additional data provided by the ASTM method for damping characterization. Castello and Matt [9] present a model building approach for transmission line cables which is guided by a pool of validation metrics. Roughly speaking, V&V encompasses several steps such as: (i) proper definition of the intended use of the model, (ii) optimum experiment design and data collection, (iii) model calibration based on parameter estimation, (iv) evaluation of the predictive capacity of the model based on new experimental data, (v) evaluation of the predictive capacity of the model

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in an environment which provides data more complex than the one used in (iii), (vi) make model re-analyses based on the result of (v), and (vii) going back to (iii) depending on the result obtained in (v) and (vi). The sequence of seven steps just presented clearly states that the procedure of calibrating a set of parameters of a chosen model based on experimental data does not assure its predictive capability. Within a V&V program, calibration can be considered as one of its essential steps.

Model calibration is accomplished based on an inverse problem formulation [10]. Concerning its predictive capabilities after calibration, they should be assessed based on quantitative comparisons between model predictions and measured quantities. The level of complexity associated with these quantitative comparisons can be increased according to a range of operational parameters and different environmental conditions [5]. The use of validation metrics [11] is a key-tool in determining the model predictive capabilities.

The present work is a continuation of the one presented by Castello et al. [8] where the authors propose a viscoelastic constitutive equation alongside an approach to estimate its parameters. This approach was assessed with data out of an experimental set-up built with a viscoelastic sandwich beam. As an effort to assess this constitutive model some preliminary analyses were performed by Borges et al. [12] using two different types of sandwich structures and by Borges et al. in [13] using a tube covered with viscoelastic sandwiched layers. In [8,12,13], all analyses were performed assuming deterministic computational models and expected values of measured data. In this work, as a continuation, we made an effort to propose a quite general approach, guided by the principles of V&V philosophy, for validation of internal variables viscoelastic models. The proposed approach takes into account the level of uncertainty of the parameters estimates to infer about model predictions [14] and uses validation metrics which rely on both measured and model uncertainties; moreover, it also considers data out of different operational scenarios [15].

The remainder of this work is organized as follows: Section 2 presents the basic characteristics of the constitutive viscoelastic model used here [8], Section 3 presents the strategy adopted for the model calibration, Section 4 presents two validation metrics [15,16] used along the validation process, Section 5 presents the results and the set of experimental set-ups that was specifically built for this work and Section 6 presents the concluding remarks.

2. Constitutive model

The modeling process itself is composed of several steps which are linked to each other through a hierarchical scheme. This process starts at the conceptual level and it gets lots of ingredients until it can be cast in a computational model that can be used in decision making environments. And here we adopt the same concept adopted by [1] for the term computational model, i.e., a numerical code which usually relies on discretization techniques and which is provided with appropriate solution algorithms. Here, our goal is to present a strategy for validation of viscoelastic models which can be used to build computational models of vibrating structures.

A survey in the literature provides several constitutive models that can be used to describe viscoelastic behavior as well as its applications in different problems such as [8,17–20], to cite a few. In the present work, we use a viscoelastic constitutive model which is built based on the thermodynamics of irreversible processes [21].

The constitutive equation used here was presented by Castello [22] and it has been used to build computational models of mechanical structures containing viscoelastic layers [8,12,13,23]. Here we will present some of its characteristics. Let us consider a viscoelastic body $\mathscr{B}$ which is under small displacements and deformation operational conditions. Its dynamic motion at each point $\boldsymbol{x} \in \mathbb{R}^d (d=1,2,3)$, and at each instant $t \in [0, t_f]$, is governed by the following equation

$$\text{div}(\boldsymbol{T}) + b = \rho \ddot{\boldsymbol{u}} \tag{1}$$

combined with constitutive equations, which here is set as

$$\boldsymbol{T} = \mathcal{N}(\dot{\boldsymbol{u}}, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^k) \tag{2}$$

in which all tensorial fields are represented in bold, $\dot{e}$ corresponds to the time derivative of a variable e and *div* denotes the divergence operator. $\boldsymbol{T}, \boldsymbol{b}$ and ρ correspond to the stress tensor, body force vector and specific mass, respectively. The linearized strain tensor is represented by $\boldsymbol{\varepsilon}$. Furthermore, the *NI* tensors $\boldsymbol{\varepsilon}^k$ represent internal variables fields [21] which are in charge of taking viscoelastic dissipation mechanisms into account. A note to be mentioned is that these hidden variables, in addition to the usual observables variables, is supposed to describe internal structures and has been found applications in viscoelasticity [8,17], elastoviscoplasticity [24], damage mechanics [21], adhesion [25], compacting fluid-saturated grounds [26], to cite a few.

Briefly speaking, the constitutive equation considering only planar deformations amenable for the applications to be pursued in this work can be suitably arranged in the matrix form presented in Eqs. (3) and (4).

$$\begin{Bmatrix} \sigma_1(\boldsymbol{x},t) \\ \sigma_2(\boldsymbol{x},t) \\ \sigma_{12}(\boldsymbol{x},t) \end{Bmatrix} = \begin{pmatrix} E_1 & E_{12} & 0 \\ E_{21} & E_2 & 0 \\ 0 & 0 & E \end{pmatrix} \begin{Bmatrix} \varepsilon_1(\boldsymbol{x},t) \\ \varepsilon_2(\boldsymbol{x},t) \\ \varepsilon_{12}(\boldsymbol{x},t) \end{Bmatrix} + \sum_{k=1}^{NI} \begin{pmatrix} E_1^{(k)} & 0 & 0 \\ 0 & E_2^{(k)} & 0 \\ 0 & 0 & E_{12}^{(k)} \end{pmatrix} \begin{Bmatrix} \varepsilon_1(\boldsymbol{x},t) - \varepsilon_1^{(k)}(\boldsymbol{x},t) \\ \varepsilon_2(\boldsymbol{x},t) - \varepsilon_2^{(k)}(\boldsymbol{x},t) \\ \varepsilon_{12}(\boldsymbol{x},t) - \varepsilon_{12}^{(k)}(\boldsymbol{x},t) \end{Bmatrix} \tag{3}$$