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Passive modifications for partial assignment of natural frequencies of mass–spring systems

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A B S T R A C T

The inverse structural modification for assigning a subset of natural frequencies of a structure to some targeted values has been found to inevitably lead to undesired changes to the other natural frequencies of the original structure that should not have been modified, which is referred to as the frequency "spill-over" phenomenon. Passive structural modifications of mass–spring systems for partial assignment of natural frequencies without frequency "spill-over" are addressed in this paper. For two kinds of lumped mass–spring systems, i.e. simply connected in-line mass–spring systems and multiple-connected mass–spring systems, two solution methods are proposed to construct the required mass-normalised stiffness matrix, which satisfies the partial assignment requirement of natural frequencies and maintains the configuration of the original structure after modifications. The modifications are also physically realisable. Finally, some examples of lumped mass–spring systems are analysed to demonstrate the effectiveness and accuracy of the proposed methods.

1. Introduction

Structural modifications (SM) are a procedure aimed at determining values of physical parameters of a structure to achieve desirable dynamic characteristics (usually modal properties such as natural frequencies and mode shapes, i.e. eigenpairs). One common task of SM is to predict modal properties as a result of structural modifications. The inverse SM problem, however, aims to determine the necessary structural modifications such that the modified structure has some prescribed desired dynamic behaviour, which usually involves an optimisation procedure looking for right modifications. As is well known, when the frequency of excitation is very close to a natural frequency excessive vibration occurs that may lead to structural failure. In this situation, it is useful to determine the changes of geometrical parameters (such as thickness, length, diameter, etc.) and/or material parameters (such as density, Young's modulus, etc.), and/or consider the addition of any combination of lumped masses and stiffnesses in order to relocate the natural frequencies concerned to other locations. This inverse structural frequency modification problem is known as frequency (eigenvalue) placement or assignment.

Mathematically, it is closely related to inverse eigenvalue problems (IEP), which involve the specification of one or more eigenvalues of a matrix or a matrix pencil and the evaluation of how the elements of the matrix need to change to result in the prescribed eigenvalues. These problems have attracted much attention of researchers over the past 30 years.

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1.1. Literature review

Research into structural modifications has been conducted mainly from two aspects: theoretical modelling (such as physical models, modal models, and frequency response function or FRF models) and experimental testing (e.g. modal testing). The goal of having desirable modal properties can be achieved by either passive or active procedures. Of course, it can be carried out by combining the above two procedures in a hybrid approach, in order to achieve the desired changes. It should be noted that even on frequency placement by structural modifications studied in this paper, there exist a large number of publications in the literature, and therefore only a brief review of some relevant papers is attempted below.

The methods for inverse structural modifications are based on the use of modal properties derived from a finite elements solution or experimental modal analysis. He [1], Sestieri and D'Ambrogio [2], and Nad [3] reviewed various structural modification methods. Tsuei and Yee [4] described a method for shifting natural frequencies by using only measured frequency response data at modification points. This is particularly convenient and effective for modal testing. Mottershead [5], and his collaborators [6,7] studied the relocation of an antiresonance and cancellation of a resonance with an antiresonance, and the assignment of natural frequencies and nodes of normal modes by the addition of grounded springs and concentrated masses using FRF data. Park and Park [8,9] used FRF formulations to find analytically the necessary multiple mass, stiffness and damping modifications in order to exactly achieve both required eigenvalues and eigenvectors. For other related works based on FRF data, refer to [10–13].

Bucher and Braun [14] derived structural changes to produce prescribed frequencies and/or mode shapes, using incomplete modal data from experimental results. Sivan and Ram [15] and Ram [16] studied the construction of a mass–spring system with prescribed natural frequencies. They [17] developed a new algorithm based on Joseph's work [18]. Gladwell [19] studied finite-element discretised structures and mass–spring structures with tridiagonal mass and stiffness matrices and derived a closed-form solution of reconstructed mass and stiffness matrices. Braun and Ram [20] analysed structures consisting of discrete masses and springs and put forward an approximate method for calculating the modification matrices of the structure.

Fox and Kapoor [21] provided expressions of both eigenvalue and eigenvector sensitivities with respect to a design parameter, which can be expressed in terms of only the corresponding unmodified modal parameters and the structure's matrices. Smith and Hutton [22] discussed the use of Newton's method and inverse iteration of mode shape updating on the frequency modification in terms of first-order expansions of eigenvalues with respect to design variables. Farahani and Bahai [23] provided algorithms for relocating eigenvalues of structures based on eigenvalue sensitivities and their second-order expansions. Djoudi et al. [24] gave a formulation free from iterations for the inverse modification of bar and truss structures. Olsson and Lidström [25] considered constraints on structures when obtaining desired frequencies. The undamped natural frequencies of a constrained structure were calculated by solving a generalised eigenvalue problem derived from the equations of motion for the constrained system involving Lagrange multipliers. Smith and Hutton [26] and Kim et al. [27] solved inverse modification problems using perturbation theory.

All these above approaches involve assigning a subset of natural frequencies of a structure to some targeted values, and inevitably lead to undesired changes to the other natural frequencies of the original structure that should not have been modified, which is referred to as the frequency "spill-over" phenomenon. For example, it may happen that an unknown frequency would gain an unwanted value, and the effects brought about by the changes in the modified structure are usually difficult to predict when a global or a large local structural modification to large-scale structures is made, because not all eigenvalues or natural frequencies of large-scale structures could be obtained accurately using the state-of-the-art techniques of matrix computations, or be measured using existing experimental facilities due to hardware limitations.

It should be mentioned that a necessary and sufficient condition was proposed for the incremental mass and stiffness matrices that modify some eigenpairs while keeping other eigenpairs unchanged in [28] but these matrices are not guaranteed to lead to physically realisable structural modifications. Additionally, there are several papers devoted to a related problem that a specific natural frequency of a structure does not change after mass and/or stiffness modifications. Çakar [29] studied a situation in which one of the pre-specified natural frequencies can be preserved by attaching a grounded spring to a structure after adding a number of masses to it. He developed a method based on the Sherman–Morrison formula in order to determine the necessary spring constant. Gürgöze and İnceoğlu [30] were concerned with satisfying a design objective such that the fundamental frequency of a cantilever beam remained the same in spite of the addition of a mass at some point on the beam. Mermertaş and Gürgöze [31] investigated the possibility of using springs to preserve the fundamental frequency of a thin rectangular plate carrying any number of point masses.

In active control of structural vibration via eigenvalue assignment techniques, the frequency "spill-over" phenomenon is overcome by using some partial eigenvalue assignment methods, which reallocate some "troublesome" eigenvalues (or natural frequencies) of the open-loop structure to suitable locations, while leaving the remaining eigenvalues and/or corresponding eigenvectors unchanged in the closed-loop structure. The partial eigenvalue assignment problem of the first-order control system has been widely studied from both theoretical and computational view points (for example, see [32,33]). To describe the dynamics of a structural system, usually a second-order differential equation is used, with structural matrices that are symmetric and sparse. However, transferring second-order equations to first-order configuration doubles the dimension of the system and the structural matrices lose some nice properties, such as positive semi-definiteness and sparsity, and even symmetry. Therefore, a large effort can be seen from the literature to have been made to tackle this problem directly on second-order dynamic system models over the past 10 years (for example, see [34–39]).