Stable force identification in structural dynamics using Kalman filtering and dummy-measurements

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ABSTRACT

Many engineering applications require the knowledge of input forces to mechanical systems. However, in practice, it is quite difficult to measure these forces directly. In order to obtain an estimate of the input forces to structural systems, Kalman filtering based techniques have recently been introduced. These state-estimation techniques allow estimating the forces concurrent with the states of a system, based on a limited number of measurements. In practice, acceleration measurements are most convenient to use in structural dynamics applications. This paper proposes an analytical analysis of the stability of the Kalman based force estimation techniques and shows that only using acceleration measurements inherently leads to unreliable results. In order to circumvent this issue, the addition of dummy-measurements on a position level is proposed. These fictitious measurements dictate the estimator to return to an undeformed state and lead to a stable estimation approach. The proposed method is validated through both a numerical and a practical experiment. Both experiments show the inadequacy of the augmented Kalman filter based on only acceleration measurements to provide stable results. The estimator with dummy measurements on the other hand provides good results in the case of an unbiased external load.

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1. Introduction

Many mechanical applications, such as condition monitoring and vibration control of both civil structures and machinery, require estimates of both the states and the inputs of structural systems. Moreover, in the majority of these cases a complete measurement of all states and inputs is infeasible. This could be due to a number of reasons:

- high cost of sensors,
- placement of sensors could compromise the structure,
- the required sensors might not even exist.

In these cases, models of the system under investigation can be used as additional sources of information to extract information from only a limited set of measurements.

In structural dynamics a typical problem is the need to estimate the input force during operational conditions. Classically this problem was solved in the frequency domain and more recently time domain methods have seen a strong rise.

An overview is given by Stevens [1], Nordström [2] and Klinkov [3]. These deterministic approaches however assume that a model is known exactly which is never the case in practice. From this point of view, stochastic methods have recently been introduced for input force estimation.

In the early sixties Kalman introduced what is now known as the *Kalman filter* (KF) [4,5]. This method provides a particularly practical and efficient state estimation algorithm for linear systems which leads to an optimal result with respect to the expected error covariance. In the original framework all inputs were assumed known, but over the years many Kalman filter based approaches for general coupled state and input estimation have been proposed in the literature [6–11]. However, the applications of these methods to structural dynamics problems and the validations of their general performance have been rather limited.

Ma et al. [12,13] presented a Kalman based approach for structural dynamics in which a Kalman filter is employed to determine the states and a separate recursive least-squares filter is employed for the input force estimation. However, this method requires a full state measurement, which might not be feasible in practice. This issue can potentially be remedied by using an adapted model [14]. Moreover, by not including the unknown input-force in the Kalman filter, bias-errors can easily arise in the results [6].

Hwang et al. [15] proposed a Kalman based approach in which the estimation of the states and external forces is coupled and which does not require a full state measurement. In this approach all model uncertainty is assumed in the unknown force. The proposed filter tuning is performed through adaptation of the measurement noise covariance, which raises questions about consistency and informativity with this approach.

Lourens et al. [16] proposed a coupled estimator based on a state-augmented Kalman filter. In this approach the regular states are augmented with the unknown forces in order to estimate them together. This approach is briefly reviewed in Section 2 and is investigated further in the remainder of this paper.[1]

In order to obtain a cost-effective, easy to implement, methodology, the methods proposed for input force estimation are based on acceleration measurements. Until now, no attention was given to the stability properties of these filters for long term estimation. This paper presents a stability analysis for augmented Kalman based algorithms in Section 3 and shows that methods based on acceleration measurements are inherently unstable. Because the error covariance estimation in the case of the augmented KF takes the relatively simple form of the Riccati equations, these stability properties can be proven analytically. It is however important to note that even though this analysis is not as straightforward for other approaches [11,17], the same stability issues might arise there.

This paper proposes to add *dummy measurements* for the positions in order to stabilize the results in Section 4. This choice was motivated due to the fact that actual position measurement would often prove impractical due to the small deformations typically associated with structural dynamics problems. The error covariance of these measurements needs to be tuned to the order of magnitude of the expected deformation of the system in order to provide good results.

The discretized version of the proposed estimator is summarized in Section 5. Finally, in Section 6 both a numerical and a physical experiment is performed to quantify the instability and validate the performance of the proposed approach with dummy measurements. These experimental results clearly show the advantage of the addition of the dummy measurements for long-term applications.

It is important to notice that the proposed approach in this work can be applied irrespective of the nature of the coordinate description of the structural model. The proposed results hold as well for regular Cartesian coordinates as for modal coordinates, shown respectively in Sections 6.1 and 6.2.

2. Augmented Kalman filter for mechanical systems

The augmented Kalman filter can be used in order to get unbiased estimates for systems with unknown inputs [7]. The basic idea is that the unknown forces are estimated together with the other states. In order to be able to perform this estimation according to a Kalman-filtering scheme, a model for the unknown forces is required as well. The system equations for this kind of system are provided in continuous time form in Section 2.1 and the continuous time KF equations are revised in Section 2.2.

2.1. Augmented state equations

The equations of motion for a structural system with external force f can be written in first order form as:[2,3]

$$\begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} + \begin{bmatrix} 0 \\ S_f \end{bmatrix} f + \begin{bmatrix} 0 \\ v_{\ddot{u}} \end{bmatrix}. \tag{1}$$

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[1] All Kalman filter properties are analyzed in the continuous time domain. This choice is made because it allows a much easier physical interpretation of the presented analytical results. All results hold equally well for discrete time Kalman filters and the validation results are obtained by using the Kalman filter in the discrete time domain.

[2] In these equations a unity mass-matrix is assumed without loss of generality.

[3] All results in this paper assume a time-invariant system, but results can be generalized to time-variant systems.