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Identification of linear systems with binary output measurements using short independent experiments

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ABSTRACT

This paper presents a novel identification technique for linear systems based on binary measurements of an analog output signal of the device under test. This type of measurement arises when the output signal is not measured directly with a high resolution sensor but it is only known whether its value exceeds a given threshold or not. The presented technique performs the estimation using a combination of many short and independent low resolution measurement runs, each of which contains only a limited amount of information. This is a significant difference with respect to the existing techniques, whose practical usefulness can be limited due to the associated requirements for long and/or periodic excitation signals. The knowledge about the binary measurement's characteristics is exploited to adapt standard prediction error minimisation techniques. This is done for both Rational Transfer Function (RTF) and Finite Impulse Response (FIR) model representations of the linear system. Simulation results are included to illustrate the identification procedure, and an experimental validation is provided to demonstrate the practical usefulness of the proposed methods.

1. Introduction

This paper presents an identification technique for linear systems with continuous amplitude input output variables whose output is measured using binary sensors. Such techniques can be used when sensors measuring the output are too costly or impractical to install. Only the knowledge about whether the output's value is above or below a given threshold is available and is used instead of its exact value. An example of a system that belongs to the prescribed class is a moving object whose position is not measured, but where an optical or magnetic sensor can detect when the object passes in front of the sensor. Other applications for such techniques are situations with indirect binary measurements where a change can be observed in the system characteristics once the threshold is passed, or situations where measurements might not be possible at all, such as software code, where only detection of threshold passings might be an option.

In all these applications, the goal is to identify a dynamic model for the system in order to predict the output for an arbitrary known input. As such, this model can be used for model-based control. If the underlying system to be modelled is Linear Time-Invariant (LTI), as assumed throughout this work, estimating the model parameters based on binary sensor information is equivalent to finding the parameters of the linear time-invariant part of the Wiener system shown in Fig. 1.

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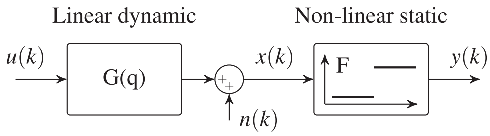

Fig. 1. Equivalent system representation consisting of a linear dynamic part and a non-linear static part representing the binary output measurement.

In this system, the non-linear static part is known and represents the binary measurement $y(k)$ of the unmeasured analog output $x(k)$ of the linear system, with

$$y(k) = F(x(k)), \tag{1}$$

where $F: \mathbb{R} \to \mathbb{R}$ equals 0 if $x(k)$ does not exceed a given, known threshold, and equals 1 if it does. Without loss of generality, the threshold's value here is arbitrarily fixed at 1, so that F is defined as

$$F(x(k)) = \begin{cases} 0 & \text{if } x(k) < 1, \\ 1 & \text{if } x(k) \geq 1. \end{cases} \tag{2}$$

The input $u(k)$ of the linear system is related to $x(k)$ by

$$x(k) = G(q)u(k) + n(k), \tag{3}$$

where q^{-1} is the delay operator such that $q^{-1}u(k) = u(k-1)$ and $G(q)$ is the system's transfer function. For the estimation, the goal is to find a discrete-time linear model $\hat{G}(q)$ that approximates the behaviour of the linear system $G(q)$ as closely as possible. This is complicated by the presence of noise $n(k)$, which is added to the output of the linear part, and represents system disturbances and measurement quantisation noise.

The presented estimation technique assumes the available data is obtained from short experiments, by which we mean each measurement only contains a single transition of the binary output. This allows one to use data measured during normal system operation (machine, process, …), such that additional identification experiments are not required. However, since only limited information is contained in each individual experiment, a unique solution can only be obtained if data from multiple, M, experiments is combined. For each of these M experiments, the system's input $u(k)$ and output $y(k)$ are collected with a sampling interval T_s, and the kth sample of the input and output during the jth experiment are denoted as $u_j(k)$ and $y_j(k)$ respectively.

Some estimation methods already exist in the literature that allow identification of systems subject to binary measurements. By considering the binary measurement as the addition of a large amount of quantisation noise to the output of a linear system, Schoukens et al. Reference [15] shows that the non-parametric best linear approximation asymptotically converges to the underlying dynamics of $G(q)$, as long as a Gaussian input $u(k)$ is used. Performing the estimation in this way however does not exploit any knowledge about the nature of the binary measurement, even though we know this in advance. As a result, long measurement times are needed to get good estimates. An additional drawback is that some restrictions are placed on the type of excitation, limiting the applicability for identification under normal machine operation.

Several methods also exist that take into account the characteristics of the non-linearity in order to speed up the estimation. In [18–20], a stochastic approach is designed specifically for the identification of systems subject to binary sensors. Periodic excitation signals are used such that the noise distribution can be estimated. With this distribution a parametric model estimate can then be derived as well that converges to the true system as the measurement time increases. While this method performs very well, the input requirements are even stricter than for [15], since the data now has to be periodic in order to estimate the noise distribution.

An alternative approach is to consider the equivalent representation of a Wiener system shown in Fig. 1. Wiener identification techniques can then be used for the estimation. However, most consider invertible and continuous non-linearities, while the binary measurement corresponds to a discontinuous non-invertible non-linearity. Another problem is that the majority of these identification techniques aim to estimate the parameters of both the linear and non-linear part of the system, whereas in this paper the non-linear part is known and this knowledge needs to be exploited to facilitate and speed up the estimation. One of the only applicable techniques is described in [10], where a general framework is developed to identify Wiener systems with non-invertible non-linearities that are known a priori. It uses the knowledge of the these non-linearities to constrain the estimated values of x, denoted $\tilde{x}$, to a number of possible allowable ranges. The values of $\tilde{x}$ are then found along with the model parameters by minimizing the difference between these $\tilde{x}$ and the $\hat{x}$ predicted by the linear model. Various non-invertible non-linearities can be handled by this method, including the case of a binary measurement. As a result, the problem formulation is general and not tuned optimally to address the case of binary measurements considered in this paper, leading to large calculation times. This method does use the data efficiently though, so our focus here will be to look for a different formulation that yields lower computation times, and which allows some further extensions. Therefore, the estimates $\tilde{x}$ and the corresponding constraints are omitted, while instead penalties are applied directly for values of $\hat{x}$ that do not correspond to the binary output. A similar method with a different formulation has been obtained in [7], but the derivation was done specifically for the case of binary sensors. The resulting optimisation problem is however still constrained, and can thus be time-consuming to solve for models other than FIR models.