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Volume-based method for spectrum sensing ☆

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ABSTRACT

It is recently shown that algorithms derived from random matrix theory (RMT) can provide superior performance for spectrum sensing, which corresponds to the task of detecting the presence of primary users in cognitive radio. The essence of the RMT-based methods is to utilize the distribution of extremal eigenvalues of the received signal sample covariance matrix (SCM), namely, the Tracy–Widom (TW) distribution. Although the TW distribution is quite useful in spectrum sensing, computationally demanding numerical evaluation is required because it does not have an explicit closed-form expression. In this paper, we devise two novel volume-based detectors by exploiting the determinant of the SCM or volume to distinguish between the signal-presence and signal-absence cases. With the use of RMT, we accurately produce the theoretical decision threshold for one of the detectors under the Gaussian noise assumption. Simulation results are included to illustrate the effectiveness of the volume-based detectors.

1. Introduction

It has been revealed in [1] that the current policies of fixed spectrum allocation do not fully utilize the available spectrum. Cognitive radio (CR) [2–12], whose main idea is to sense the spectrum over a wide range of frequency bands and exploit the temporally unoccupied bands for opportunistic wireless transmissions, is a promising paradigm to increase the spectrum usage efficiency. In a CR network, when the spectrum resources of a primary user (PU) are not occupied, a secondary user (SU) is allowed to use them. That is to say, the SU needs to reliably detect the presence of the PU. This is referred to as spectrum sensing, which can be cast as a binary hypothesis testing problem and is particularly challenging for small sample size and/or low signal-to-noise ratio (SNR) conditions.

For the scenario of signal-absence, the observed data only consist of noise and are usually assumed to be independent and identically distributed (IID). It is apparent that the energy and correlation structure of the observations differ when the PU signal is present. As a result, spectrum sensing can be achieved by making use of these dissimilarities. When the noise power is known, the energy detector (ED) [13,14] has been shown to be optimal for the IID PU signals. However, the noise power information is usually unavailable in practice and thus its estimate is used instead [15–17]. This can dramatically degrade the detection performance of the ED approach because it is quite sensitive to the noise uncertainty. As a matter of fact, the eigenvalues of the received signal sample covariance matrix (SCM) in the signal-presence situation are more spread out than those in the noise-only case, which corresponds to a scaled identity matrix in the asymptotic sense. The spread-out eigen-spectrum results from the correlation structure inherent in the covariance matrix. As a result, a number of eigenvalue-based detectors which exploit the correlation structure for spectrum sensing have been proposed in the literature [18–25]. Derived in the framework of generalized likelihood ratio test (GLRT), the arithmetic-to-geometric mean (AGM) method [18] is able to reliably identify the correlated signals embedded in the IID noise. However, the AGM algorithm has its root in the maximum likelihood (ML) theory which turns out to be inefficient when the temporal and spatial dimensions are small, that is, the theoretical decision threshold cannot be accurately determined. On the other hand, the maximum-to-minimum eigenvalue (MME) approach [19] is heuristically developed to test if the SCM corresponds to an identity matrix or its correlated alternative with the use of its maximum and minimum eigenvalues. Since not all eigenvalues are utilized, its detection performance is highly sensitive to weak correlated signals and/or small sample sizes. Moreover, computation of the theoretical threshold for the MME algorithm

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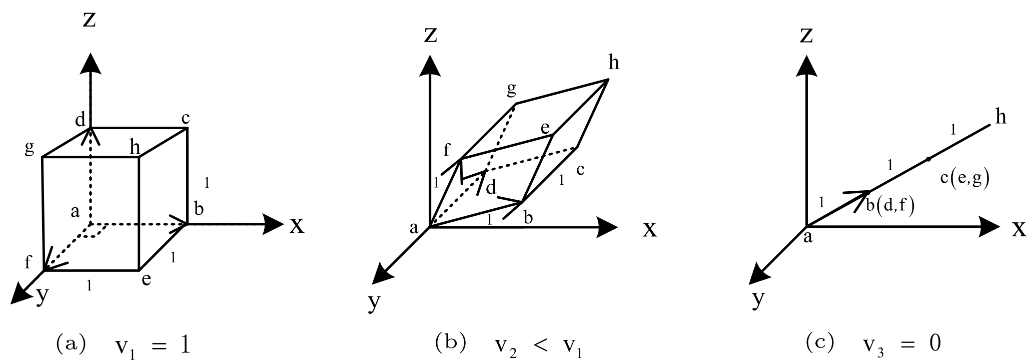

Fig. 1. Volume comparison for uncorrelated, correlated and coherent observations.

relies on the distribution of the maximum and minimum eigenvalues in the framework of random matrix theory (RMT), namely, the Tracy–Widom (TW) distribution [26]. However, there is no explicit closed-form expression for the TW distribution, indicating that an additional overhead of numerical evaluation is required. For the situation where there is only a single primary signal, an accurate variant of the GLRT has been devised for spectrum sensing in [24], which is equivalent to the signal-to-noise (mean) eigenvalue (SNE) method [21]. As discussed in [27,30], nevertheless, the number of primary signals in the sensed channel can be more than one. Under such a condition, the performance of the SNE method cannot be guaranteed. In practice, the SU receivers are usually uncalibrated, making the noises at different antennas to be non-uniform. To handle the non-uniform noise, some robust sensing approaches have been proposed, such as the GLRT test [28], independence test [29], Hadamard ratio test [27,30] and locally most powerful invariant test (LMPIT) [31]. In this work, a new philosophy for spectrum sensing is devised to accurately and robustly detect the PUs in a computationally attractive manner. The underlying idea is that the determinant of SCM or volume differs dramatically between the signal-absence and signal-presence situations.

The rest of the paper is organized as follows. The problem formulation of spectrum sensing is presented in Section 2. In Section 3, prior to deriving the volume-based detectors, the motivation is provided via geometric interpretation. Then two volume-based detectors, denoted by VD1 and VD2, are developed for spectrum sensing. With the use of RMT, the theoretical decision threshold of the VD2 is accurately determined and no numerical procedure is involved. Simulation results are included in Section 4 to evaluate the performance of the proposed detectors by comparing with the ED, AGM, MME, Hadamard ratio and SNE methods. Finally, conclusions are drawn in Section 5.

2. Problem formulation

Consider a multipath fading channel model and assume there are 1 PU and $(d-1)$ interference users with $d \geqslant 1$, and each of them is equipped with a single antenna in a CR network. To simplify the following presentation, the interference users are now counted as PUs because they occupy the same channel, that is, there are d PUs. To find a temporally unoccupied channel, a SU receiver with m antennas needs to monitor this channel. Denote the signal-absence and signal-presence hypotheses by $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The output observations of the SU, $\boldsymbol{y}(k)$ $(k=1,\ldots,n)$, under the binary hypotheses can be written as

$$\boldsymbol{y}(k)=\begin{cases}\boldsymbol{w}(k), & \mathcal{H}_0\\ \boldsymbol{H}\boldsymbol{s}(k)+\boldsymbol{w}(k), & \mathcal{H}_1\end{cases} \tag{1}$$

where n is the number of samples, $\boldsymbol{H}\in\mathbb{R}^{m\times d}$ represents the fading channels between the PUs and SU, and

$$\boldsymbol{y}(k)=\big[x_1(k),\ldots,x_m(k)\big]^T \tag{2}$$

$$\boldsymbol{s}(k)=\big[s_1(k),\ldots,s_d(k)\big]^T \tag{3}$$

$$\boldsymbol{w}(k)=\big[w_1(k),\ldots,w_m(k)\big]^T \tag{4}$$

stand for the observation, signal and noise vectors, respectively, with $(\cdot)^T$ being the transpose operator. Unless stated otherwise, the channels, primary signals and noise are considered to be real-valued[1] throughout this paper. We assume that the noises are statistically independent and satisfy $w_i(k)\sim\mathcal{N}(0,\sigma_{w_i}^2)$ $(i=1,\ldots,m)$ where $\sigma_{w_i}^2$ is the unknown noise variance, $\sim$ represents “distributed as” and $\mathcal{N}(\mu,\Sigma)$ denotes the Gaussian distribution with mean μ and variance Σ. If $\sigma_{w_i}^2=\sigma_w^2$ for $i=1,\ldots,m$, the noise becomes IID (uniform); otherwise, it is the non-uniform noise due to the uncalibrated receiver [28,32]. Meanwhile, suppose that $s_i(k)$ $(i=1,\ldots,d)$ is a random process with mean zero and unknown variance $\sigma_{s_i}^2$, which is independent of the noise. Note that the primary signal vector $\boldsymbol{s}(k)$ is unnecessarily Gaussian distributed. In order to exploit the correlation structure inherent in the observations, we employ the covariance matrix of $\boldsymbol{y}(k)$, given as

$$\boldsymbol{R}=\mathbb{E}\big[\boldsymbol{y}(k)\boldsymbol{y}^T(k)\big] \tag{5}$$

where $\mathbb{E}[\cdot]$ is the expectation operator.

3. Volume-based detector for spectrum sensing

3.1. Geometric interpretation

The determinant of $\boldsymbol{R}$ in fact is the hyper-volume of the geometry determined by the row vectors of $\boldsymbol{R}$. As an example, let us consider the scenario of three receiving antennas where the observed data with zero mean and unity variance may be independent, correlated or coherent. This means that the corresponding covariance matrices are the 3×3 identity matrix, full-rank non-identity matrix and rank-one arbitrary matrix. The geometries, namely, cube, parallelepiped and line, formed by the row vectors of the matrices are depicted in Fig. 1, where all edges of the geometries are assumed to be unity such that $\|\boldsymbol{R}(i,:)\|=1$ with $\boldsymbol{R}(i,:)$ being the i-th row of $\boldsymbol{R}$ and $\|\cdot\|$ being the Euclidean norm. Here, the volumes of the cube, parallelepiped and line, are denoted by v_1, v_2 and v_3, respectively. The cube corresponds to the case of signal-absence whereas the other two geometries are referring to the cases of signal-presence. For the signal-absence situation, the covariance matrix is a 3×3 identity matrix, i.e., $\boldsymbol{R}=\boldsymbol{I}_3$, whose rows determine the coordinates of the points b, f and d in Fig. 1(a), that is, $(x_b,y_b,z_b)=(1,0,0)$, $(x_f,y_f,z_f)=(0,1,0)$, $(x_d,y_d,z_d)=(0,0,1)$. Consequently, we obtain $\mathrm{v}_1=1$. For the

[1] The proposed methods can be readily applied to the complex-valued case by transforming the complex observation to its real counterpart, see [33] for example.