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Failure detection with likelihood ratio tests and uncertain probabilities: An info-gap application

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ABSTRACT

The likelihood ratio test (LRT) has attractive failure-detection properties. However, evaluating the likelihood ratio and implementing the LRT require knowledge of the underlying probability distributions. Data or knowledge, especially about future failures, is often quite limited. In this paper we employ the info-gap robustness function in specifying the parameters of the LRT when the probability distributions are imperfectly known. We develop an info-gap analog of the probabilistic detection-error trade off curve, and demonstrate the results by application to pressure measurements on an industrial production device.

1. Introduction

Failure detection is the discovery of the occurrence of anomalous behavior, as distinguished from failure diagnosis that characterizes the nature and origin of the anomaly. We employ info-gap theory in formulating a failure detection algorithm based on the likelihood ratio test (LRT), given imperfect knowledge of the probability distribution of future failures.

Failure detection has been studied systematically and in depth at least since the early decades of the 20th century, and is the subject of numerous books and articles studying both theory and application. This literature applies a wide range of tools from statistics and signal processing. For instance, Gertler [16, Chapter 11] applies statistical tests on residuals (the difference between measurements and model predictions) to detect the presence of failure. Patton et al. [23] discuss a range of filter and observer techniques for failure detection. Williams [29] summarizes model-based failure detection. Willsky [30,31] discusses likelihood ratio methods for detection of abrupt changes. Pau [24, Chapter 4] explores time-minimization statistical tests. Basseville and Nikiforov [2] provide a masterful and very accessible discussion of statistical tools—especially based on the likelihood ratio concept—for detecting abrupt changes in dynamical systems. Campbell and Nikoukhah [8]

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explore interrogative methods—applying external signals—for failure detection. We mention just a few of the myriad recent specific applications. Tibaduiza et al. [27] build statistical models of dynamic behavior for detecting and classifying damage in structural health monitoring. Hwang et al. [19] develop a condition monitoring method based on likelihood change of a stochastic model of the system in normal operation. Earls [13] uses non-contact and very sparse contact inspection methods for non-destructive evaluation and testing for hidden corrosion in steel bridge connections.

Much effort has been invested in studying the relations among these various approaches to failure detection. Quite often seemingly distinct methods—such as likelihood ratio tests and residual parity checks—are equivalent in the sense that either method can be represented by the other [2, p. 241]. Nonetheless, representationally equivalent methods may be very different in computational difficulty or in their requirements for prior information. The choice of a failure detection strategy depends on the type of system, the type of failure, and the available information.

The LRT has an optimality property that makes it very attractive: minimal probability of missed detection (type II error), for given probability of false alarm (type I error). However, evaluating the likelihood ratio and implementing the LRT require knowing the underlying probability distributions. Data or knowledge, especially about future failures, is often quite limited. In this paper we employ the info-gap robustness function to specify the parameters of the LRT when the underlying probability distributions are imperfectly known.

Robustness to uncertainty is a central concept in many approaches to failure detection. Chow and Willsky [9], Gertler [15], Frank [14] and many others discuss methods of 'analytical redundancy' and 'residual generation' for exploiting redundant information in system models to enhance robustness against modeling error. Ding et al. [12] continue this direction and discuss observer-based methods for robustly detecting unknown inputs to linear time-invariant systems.

'Robustness' has many meanings. The concept of robustness used in this paper derives from a prior concept of non-probabilistic uncertainty. Knight [20] distinguished between 'risk' based on known probability distributions and 'true uncertainty' for which probability distributions are not known. Similarly, Ben-Tal and Nemirovski [7] are concerned with uncertain data within a prescribed uncertainty set, without any probabilistic information. Likewise Hites et al. [17, p. 323] view "robustness as an aptitude to resist to 'approximations' or 'zones of ignorance'", an attitude adopted also by Roy [26]. We are also concerned with robustness against Knightian uncertainty. We consider uncertainty in probability distributions but we do not pursue an explicitly statistical approach to robustness as studied by Huber [18] and many others.

Our approach is in the tradition of Wald. Wald [28] studied Knightian uncertainty in the problem of statistical hypothesis testing based on a random sample whose probability distribution is not known, but whose distribution is known to belong to a given class of distribution functions. Wald states that "in most of the applications not even the existence of … an a priori probability distribution [on the class of distribution functions] … can be postulated, and in those few cases where the existence of an a priori probability distribution … may be assumed this distribution is usually unknown." (p. 267).

In this paper we quantify Knightian uncertainty using info-gap theory ([3]; info-gap.com). Info-gap theory has been applied to various problems of statistical inference where probabilistic properties or distributions are incompletely known. Pierce et al. [25] use info-gap theory in assessing the reliability of artificial neural nets for damage detection. Zacksenhouse et al. [32] use info-gap theory in managing data uncertainty in linear regression for neural decoding of brain–machine interfaces. Ben-Haim [4] employs info-gap theory in regression of economic data and for confidence interval estimation given uncertain probabilities. Mirer and Ben-Haim [22] use info-gap theory in the design and analysis of 'penalty tests' in which excess stresses are applied to an explosive material in assessing its safety against accidental actuation and its reliability of operational actuation. Ben-Haim [5] applies info-gap theory in statistical evaluation of null results—not detecting the presence of a pernicious agent—when the degree of statistical correlation between observations is uncertain.

The basic properties of the LRT are discussed and illustrated in Sections 2 and 3. The robustness function is formulated in Section 4, and demonstrated on industrial data in Section 5. Specification of the parameters needed for implementation of the LRT algorithm and use of the robustness function in assessing confidence in the decision are illustrated in Section 6. We also demonstrate an info-gap analog of the probabilistic detection-error trade off curve (also known as the Receiver Operating Characteristic, or ROC).

2. Statistical concepts and hypothesis tests

In this section we define the requisite statistical concepts and notation.

Probability distributions and innovations: Our task is to detect the occurrence of change in the system. Let θ be a vector of parameters that specifies the system, where θ_0 and θ_1 are the values before and after the onset of failure, respectively. We will assume that θ_0 is known from system identification in failure-free conditions, while θ_1 is unknown or highly uncertain.

We assume that we have access to the inputs and outputs of the system. A set of consecutive control inputs is denoted $Z_i^j = \{z_i, \dots, z_j\}$ at time steps $i, i+1, \dots, j$. A set of consecutive output measurements is denoted $X_i^j = \{x_i, \dots, x_j\}$.

The measured output vector x_i is a random variable whose probability density function (pdf) is denoted $p(x|\theta)$. Thus x is distributed according to $p(x|\theta_0)$ before failure, and according to $p(x|\theta_1)$ after failure. We will assume that $p(x|\theta_0)$ is known or reliably estimated. In contrast, we are uncertain about some aspects of $p(x|\theta_1)$.

Given measured inputs Z_1^i and outputs X_1^{i-1} and a specification θ of the system we can construct the conditional pdf of the next output, x_i, denoted $p(x_i|Z_1^i, X_1^{i-1}, \theta)$. This may employ system identification, or state-estimators such as the Kalman filter. If $i=1$, so that x_1 is the first output, we define $p(x_1|Z_1^1, X_1^0, \theta)$ as $p(x_1|z_1, \theta)$. We can write the joint pdf of the outputs X_1^m