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Uncertainty law in ambient modal identification—Part II: Implication and field verification

Siu-Kui Au [a,b,*,1]

[a] Center for Engineering Dynamics, University of Liverpool, United Kingdom
[b] Institute for Risk and Uncertainty, University of Liverpool, United Kingdom

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ABSTRACT

This paper presents a qualitative analysis of the uncertainty laws for the modal parameters identified in a Bayesian approach using ambient vibration data, based on the theory developed in the companion paper. The uncertainty laws are also appraised using field test data. The paper intends to provide insights for planning ambient vibration tests and managing the uncertainties of the identified modal parameters. Some typical questions that shall be addressed are: to estimate the damping ratio to within 30% of posterior coefficient of variation (c.o.v), what is the minimum data duration? Will deploying an additional accelerometer significantly improve the accuracy in damping (or frequency)? Answers to these questions based on this work can be found in the Conclusions. As the Bayesian approach allows full use of information in the data for given modeling assumptions, the uncertainty laws obtained in this work represent the lower limit of uncertainty (estimation error) that can be achieved by any method (Bayesian or non-Bayesian).

1. Introduction

Uncertainty in the identified modal parameters is an important aspect to manage in planning an ambient vibration test. Channel noise, sampling rate and bandwidth of data can be well-controlled nowadays, thanks to advances in modern sensing and data acquisition technology. The attributes that often need to be decided on a case-by-case basis include, among others, the number of sensors, the location of sensors and the data duration. Mechanical concepts, together with experience with sensing and data acquisition hardware, can help configure these attributes. The number of sensors is constrained by availability and budget. The location of sensors depends on the mode shapes expected to be found. Logistics and accessibility constraints are critical factors, although progress in theoretical development cannot be overlooked [1–4]. The duration of data is often decided by rule of thumb, e.g., 1000 natural periods of the lowest mode of interest. It can be constrained by the available time, e.g., on a construction site. In principle, increasing the data length is expected to improve accuracy by virtue of increasing the amount of information. Identifying parameters using an extended data length, however, can increase modeling error risk [5]. For example, assuming a time-invariant model, the damping ratio identified based on a long period of data where the response amplitude has changed significantly can at best represent the average value of the actual amplitude-dependent damping [6–9]. The damping ratio is an important parameter in applications as it directly affects the magnitude of dynamic response. However, there is no commonly accepted method for reliable prediction at the

* Correspondence to: School of Engineering, Harrison Hughes Building, Brownlow Hill, L69 3GH, Liverpool, United Kingdom. Tel.: +44 151 794 5217.
E-mail addresses: siukuiau@gmail.com, siukuiau@liverpool.ac.uk
[1] Formerly City University of Hong Kong.

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design stage. It is also difficult to estimate from measured data, due to, e.g., measurement error, modeling error and amplitude-dependence. Methods that rely on statistical proxies (e.g., averaging) are vulnerable to bias [10–13]. It is necessary to quantify the uncertainty associated with damping estimates so that the results can be interpreted in the right context.

A Bayesian FFT approach allows full extraction of information contained in the data for modal identification [14]. The raw FFTs instead of their averaged counterparts are used for statistical inference, therefore eliminating possible distortion due to averaging or other signal processing artifacts. Based on the same data and modeling assumptions, no non-Bayesian method can be more informative about the modal parameters than the Bayesian method. The uncertainty laws therefore represent the lower limit of estimation error that can be achieved by any method (Bayesian or non-Bayesian) for given data and modeling assumptions.

An asymptotic analysis of the 'posterior uncertainty' (i.e., given data) of modal parameters in a Bayesian context has been performed in the companion paper. Assuming well-separated modes, small damping and sufficient amount of data, asymptotic expressions for the posterior covariance matrix of modal parameters have been derived. The results are *remarkably simple*. This paper presents a qualitative analysis of the uncertainty laws to yield insights for planning ambient vibration tests and managing the uncertainties of the identified modal properties. The uncertainty laws are also verified using field test data.

2. Qualitative analysis

We first recall the main results derived in the companion paper. To the leading order, the (squared) posterior coefficient of variation (c.o.v. = standard deviation/most probable value) of the natural frequency f, damping ratio ζ, PSD (power spectral density) of modal force S, and PSD of prediction error S_e, are given by

$$\delta_f^2 \sim \frac{\zeta}{2\pi N_c B_f(\kappa)}, \quad \delta_\zeta^2 \sim \frac{1}{2\pi\zeta N_c B_\zeta(\kappa)}, \quad \delta_S^2 \sim \frac{1}{N_f B_S(\kappa)}, \quad \delta_{S_e}^2 \sim \frac{1}{(n-1)N_f} \tag{1}$$

where n is the number of measured degrees of freedom (dofs); $N_c = T_d/T$ (T_d = data duration; T = natural period) is the data length as a multiple of the natural period; $N_f = 2\kappa\zeta N_c$ is the number of frequency ordinates in the selected band $f(1 \pm \kappa\zeta)$ and κ is the 'bandwidth factor';

$$B_f(\kappa) = \frac{2}{\pi}\left(\tan^{-1}\kappa - \frac{\kappa}{\kappa^2+1}\right), \quad B_\zeta(\kappa) = \frac{2}{\pi}\left[\tan^{-1}\kappa + \frac{\kappa}{\kappa^2+1} - \frac{2(\tan^{-1}\kappa)^2}{\kappa}\right]$$
$$B_S(\kappa) = 1 - 2(\tan^{-1}\kappa)^2\kappa^{-1}\left(\tan^{-1}\kappa + \frac{\kappa}{\kappa^2+1}\right)^{-1} \tag{2}$$

are 'data length factors' that depend only on κ.

The bandwidth factor κ is a dimensionless parameter that depends on the frequency band selected by the user, which must trade off between modeling error and the information included for modal identification. Theoretically, the wider the selected band (hence larger κ) the more information for identification. However, widening the band makes the identification model more vulnerable to modeling error regarding single mode and constant PSD of modal force/prediction error within the band.

The posterior covariance matrix of the mode shape $\boldsymbol{\Phi} \in R^n$ (with normalization $\|\boldsymbol{\Phi}\|^2 = \boldsymbol{\Phi}^T\boldsymbol{\Phi} = 1$) is given by

$$\mathbf{C}_\Phi \sim \frac{\nu\zeta}{N_c B_\Phi(\kappa)}(\mathbf{I}_n - \boldsymbol{\Phi}\boldsymbol{\Phi}^T) \tag{3}$$

where $\nu = S_e/S$ is the 'noise-to-environment (n/e) ratio'; and

$$B_\Phi(\kappa) = \tan^{-1}\kappa \tag{4}$$

is the data length factor for the mode shape. The expected Modal Assurance Criterion (MAC) that quantifies the overall uncertainty of the mode shape [15] is given by

$$\overline{\rho} = (1 + \delta_\Phi^2)^{-1/2} \tag{5}$$

where δ_Φ^2 is the sum of principle variances of $\mathbf{C}_\Phi$ given by

$$\delta_\Phi^2 \sim \frac{(n-1)\nu\zeta}{N_c B_\Phi(\kappa)} \tag{6}$$

2.1. Governing scales

The posterior uncertainties in (1) and (3) depend on the following (dimensionless) scales: ζ, ν, κ, N_c or N_f. The damping ratio ζ is a property of the structure. The n/e ratio ν represents a modal noise-to-signal ratio excluding the effect of dynamic amplification. The 'normalized' data length N_c is related to the maximum amount of information available in the data for