Mechanical Systems and Signal Processing 48 (2014) 232–246

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Theory of un-scaled flexibility identification from output-only data

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A R T I C L E I N F O

Article history:
Received 27 November 2013
Received in revised form
5 February 2014
Accepted 12 February 2014
Available online 11 March 2014

Keywords:
Flexibility
Scaling
FRF magnitude
Output-only
Modal identification

A B S T R A C T

The magnitudes of the frequency response functions (FRFs) from ambient vibrations are seldom to be investigated because they are not necessary for basic modal parameter (frequency, damping and mode shape) identification. However, they are necessary for structural flexibility identification. In this article, the FRF magnitudes from output-only data are investigated, and an approach to identify structural un-scaled flexibility characteristics is proposed. It first quantitatively formulates the relation between the magnitudes of the FRFs estimated from the vibration test data with known/unknown input forces. It is proved that the magnitude ratios between the above two kinds of FRFs are mode-dependent. Then, a way to calculate the magnitude ratios is proposed to scale the FRFs from output-only data. Finally, the closed-form solution of the un-scaled flexibility is derived from ambient vibration data. The identified un-scaled flexibility will reveal the load–deflection characteristics of a structure to further engineers' understanding about the structure's safety condition. Numerical and experimental examples illustrate the effectiveness of the un-scaled flexibility identification theory.

1. Introduction

A variety of methods have been developed to process output only data for structural modal parameter identification, and they have been widely applied to a number of real bridges [1–5]. Basic modal parameters including frequencies, damping ratios and mode shapes have direct relations with structural properties (mass, stiffness, boundary, connection etc.), however, directly utilizing them into engineering practices for structural management is still a challenging problem. A few studies have been made to directly identify the stiffness/flexibility matrix from output-only data. For instance, Koh et al. developed a series of GA-based (Genetic Algorithm) method to search the stiffness values for best fitting the measurement. Filtering technologies including the Kalman filter [6], H-infinite filter [7], and Monte Carlo filter [8] were proposed to identify structural stiffness by writing the motion equation into the state-space form. These methods together with others [9–12] significantly contributed to structural stiffness identification even in the time-varying and nonlinear cases [13–15], however, they have not been widely applied in engineering practices due to their computation efficiency and other reasons.

Impact testing with the input force measurement has the merit to extract not only dynamic (modal parameters) but also static (flexibility) characteristics of a structure. Because both input forces and output responses are measured, the frequency

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http://dx.doi.org/10.1016/j.ymssp.2014.02.009
0888-3270

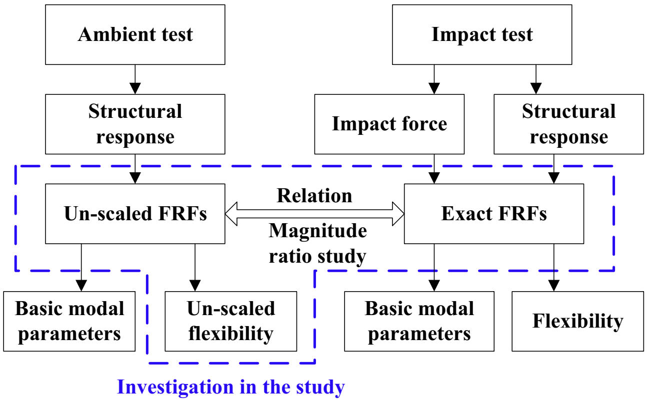

Fig. 1. Concept of the un-scaled flexibility identification.

response function (FRF) estimated from the impact test data is exactly the same as the analytical one of the studied structure, from which structural modal flexibility can be accurately identified [16–18]. However, the following reasons hinder the widespread applications of the impact testing method into engineering practices: (1) it requires traffic control during the field test thus it leads to expensive experiment cost; (2) the impacting force should have a huge magnitude and a wide frequency range, while the available excitation equipments are limited; and (3) the current impact testing technology mainly works for short/middle span bridges because long-span bridges are difficult to be excited by human-made forces.

The theory of un-scaled flexibility identification from the output-only data will be developed in this article. Here the un-scaled flexibility is defined as the one that is proportional to the exact flexibility with a constant scaling ratio. The key issue to identify the flexibility characteristics from output-only data is studying the magnitudes of the estimated FRFs. Both impacting forces and structural responses are measured during the impact test, thus they output the exact FRFs of the investigated structure. Namely, the magnitudes of the estimated FRFs from the impact test data are exactly the same as the analytical values calculated from structural inherent parameters (mass, stiffness, and damping). In contrast, the magnitudes of the FRFs from output-only data are different with the analytical values. Numerous articles are available comparing the FRFs estimated from the impact test and ambient test data [19], but almost all of them did not care about the FRF magnitudes because their purposes were only to identify basic modal parameters (frequency, mode shape, and damping), while the FRF magnitudes are necessary for structural flexibility identification.

In this article, the writers first investigate the FRF magnitudes from output-only data by deriving its close-form expression from the basic equation of motion (Fig. 1). It is found that its magnitudes are proportional to exact FRF magnitudes in each mode, but the magnitude ratio is mode-dependent. It is also found that the magnitude ratio is dependent on the reference node, while it is independent with the output node. Subsequently, the method to calculate the magnitude ratios is proposed and the calculated magnitude ratios are used to scale the FRFs from output-only data. Based on the above findings, the theory to identify the un-scaled flexibility is developed. Numerical and experimental examples are investigated to illustrate the findings described above and to verify the effectiveness of the proposed method for un-scaled flexibility identification. Finally, conclusions are drawn.

2. Investigation of the FRF magnitudes

2.1. Closed-form expression of the FRFs

The motion equation of an n-Degree of Freedom (DOF) structure in the modal space is [20]

$$\ddot{\boldsymbol{q}}^{\boldsymbol{r}}(\boldsymbol{t})+2^{r}\omega_{n}^{r}\dot{\boldsymbol{q}}^{\boldsymbol{r}}(\boldsymbol{t})+\omega_{n}^{r2}\boldsymbol{q}^{\boldsymbol{r}}(\boldsymbol{t})=\frac{1}{M^{r}}\{\boldsymbol{\varphi}^{\boldsymbol{r}}\}^{\boldsymbol{T}}\boldsymbol{f}(\boldsymbol{t}) \tag{1}$$

where ω_n^r = the rth modal frequency, ξ^r = the rth modal damping ratio, $\boldsymbol{\varphi}^{\boldsymbol{r}}$ = the rth mode shape, M^r = the rth modal mass, and $\boldsymbol{f}(\boldsymbol{t})$ = the input force. The structural response at the node o excited by the input force at the node i is solved using Duhamel integration,

$$\boldsymbol{x}_{\boldsymbol{oi}}(\boldsymbol{t})=\sum_{r=1}^{n}\varphi_{\boldsymbol{o}}^{\boldsymbol{r}}\varphi_{\boldsymbol{i}}^{\boldsymbol{r}}\int_{-\infty}^{\boldsymbol{t}}\boldsymbol{f}_{\boldsymbol{i}}(\tau)\boldsymbol{g}^{\boldsymbol{r}}(\boldsymbol{t}-\tau)d\tau \tag{2}$$

where $\boldsymbol{g}^{\boldsymbol{r}}(\boldsymbol{t})=\frac{1}{M^r\omega_d^r}\exp(-\xi^r\omega_n^r\boldsymbol{t})\sin(\omega_n^r\boldsymbol{t})$, $\omega_d^r=\omega_n^r(1-\xi^{r2})^{1/2}$, $\boldsymbol{\varphi}_{\boldsymbol{o}}^{\boldsymbol{r}}$ and $\boldsymbol{\varphi}_{\boldsymbol{i}}^{\boldsymbol{r}}$ are the rth mode shape values at the oth and the ith nodes respectively. When the input force, $\boldsymbol{f}(\boldsymbol{t})$, is a Dirac delta function, the impulse response function (IRF) of the structure