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Accurate frequency domain measurement of the best linear time-invariant approximation of linear time-periodic systems including the quantification of the time-periodic distortions

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ABSTRACT

Time-periodic (TP) phenomena occurring, for instance, in wind turbines, helicopters, anisotropic shaft-bearing systems, and cardiovascular/respiratory systems, are often not addressed when classical frequency response function (FRF) measurements are performed. As the traditional FRF concept is based on the linear time-invariant (LTI) system theory, it is only approximately valid for systems with varying dynamics. Accordingly, the quantification of any deviation from this ideal LTI framework is more than welcome. The “measure of deviation” allows us to define the notion of the best LTI (BLTI) approximation, which yields the best – in mean square sense – LTI description of a linear time-periodic LTP system. By taking into consideration the TP effects, it is shown in this paper that the variability of the BLTI measurement can be reduced significantly compared with that of classical FRF estimators. From a *single* experiment, the proposed identification methods can handle (non-)linear time-periodic [(N)LTP] systems in open-loop with a quantification of (i) the noise and/or the NL distortions, (ii) the TP distortions and (iii) the transient (leakage) errors. Besides, a geometrical interpretation of the BLTI approximation is provided, leading to a framework called *vector FRF analysis*. The theory presented is supported by numerical simulations as well as real measurements mimicking the well-known mechanical Mathieu oscillator.

1. Introduction

Many systems from different engineering fields exhibit a (quasi) cyclic behavior over time. Think of blade-to-blade manufacturing defects in wind turbines [1], twisted-actuated helicopter rotor blades during forward flight [2], anisotropic shaft-bearing systems in rotating machinery [3], (periodic) moving mass distribution [4], electrical impedance measurements of a living heart for cardiovascular monitoring [5] or even nonlinear time-invariant (NLTI) systems being linearized around a periodically varying operation point [6,7] to name a few. Those time-periodic (TP) observations are often not taken into account when traditional frequency response function (FRF) measurements are conducted.

FRF measurements are known to be a very valuable and simple tool for characterizing the dynamical behavior of systems. They can be found in all kinds of engineering disciplines (bio-medical, mechanical, acoustical, civil, electronic, electro-chemical, heat

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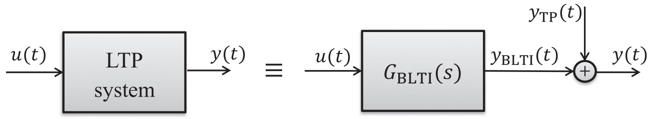

Fig. 1. Best LTI (BLTI) approximation $G_{\mathrm{BLTI}}(s)$ of an LTP system with input $u(t)$ and output $y(t)$. The time-periodic (TP) distortion $y_{\mathrm{TP}}(t)$ is that part of the actual output $y(t)$ that cannot be explained by the BLTI model.

transfer, etc.) for getting quick insight into the dynamics of the complex system under test [8–14,5]. FRF measurements also often used as a validation test tool in parametric estimation problems [8].

As the classical concept of an FRF $G(j\omega) = G(s)|_{s=j\omega}$ is associated with a linear time-invariant (LTI) transfer function $G(s)$, it should be justified in practice whether the time-invariance and linearity assumption, which is inherent to $G(s)$, are really met by the system under test. While the impact of nonlinear (NL) distortions on FRF measurements has thoroughly been investigated during the last decade [8,10,15–17], the influence of time-variant (TV) effects on FRF measurements has only been examined recently for slowly arbitrarily TV systems [18–20].

This work focuses on a particular class of TV systems, namely linear time-periodic (LTP) systems, where we attempt to find answers to the following questions: (i) Does it make sense to approximate the description of the dynamics of LTP systems by an LTI model? (ii) How large is the error one makes using such an approximation? (iii) Can simple tools be set up to visualize in an easy way the errors due to this approximation? To this end, the LTP system is decomposed into an LTI part, which will be denoted as the best linear time-invariant (BLTI) approximation, the output of which is disturbed by an additive error term caused by the TP part of the system (see Fig. 1). In that frame of mind, the BLTI framework is useful to assess the validity of LTI models used, for instance, in applications where prediction, control or physical interpretation are of interest. It turns out that, when FRF measurements are made, a stochastic "noise" error is introduced due to the time-variation. Depending on the degree of the time-variation, the error can be surprisingly significant, which is often wrongly attributed to noise [18]. Therefore, it is desirable to know the order of magnitude of the error due to time-variation, to noise and/or to NL distortions such that appropriate actions can be taken by the user. As an example, if the predictive power of the BLTI model for control is accurate enough, then the LTI control theory might be sufficient instead of relying on more difficult (N)LTP/(N)LTV control strategies [21].

Two existing nonparametric methods, the spectral analysis method (SAM) [9] and the local polynomial method (LPM) [22], for measuring the BLTI approximation are presented and compared with each other. It is shown here that the BLTI approximation can be measured with a reduced uncertainty compared with that of traditional FRF estimators. Besides, a clear distinction can be made between the noise errors, the leakage (transient), the TP distortions and the total distortions (such as NL distortions, unmodeled time-variations and bias errors) when a synchronized *periodic* excitation is applied. The drawback of using a *non*-periodic stimulus is that one cannot discriminate between the noise and the NL distortions [8,23].

The main differences with the recent contributions reported in [18–20,24] are

- We now explicitly make use of the periodicity condition of LTP systems such that the computation of the numerical derivative over the frequency is avoided when estimating $G_{\mathrm{BLTI}}(s)$. The framework in [18–20] is developed for slowly arbitrarily TV systems.
- The LPM suggested in [24] is extended here to handle *fast varying* LTP systems in transient regime as well. An LTP system is called fast varying if its memory length is much longer than the periodicity T_{sys} of the LTP system [23] (see Fig. 7 (top, right) for an illustration).

Fast varying (N)LTP systems have been covering a wide range of applications where systems sustain a periodic motion (e.g. wind turbines, helicopter blades, anisotropic shaft-bearings systems, combustion engines, etc.) [2,3,6,25,26]. Few frequency domain parametric and nonparametric estimators exist for fast varying LTP systems in the literature. The SAM of Bendat and Piersol [9] has been extended by Siddiqi [27] to estimate the *harmonic* FRFs (*h*-FRFs) of LTP systems during steady state by using chirp signals as test signals. The same frequency domain method has been applied on the vibration measurements of helicopter blades during forward flight in [2]. However, no uncertainty bounds are given for the measured quantities. Since the uncertainty of a(n) (harmonic) FRF is as important as the FRF itself [13], we will extend the SAM in [2] to the transient regime including an uncertainty analysis.

On the other hand, Allen [25] proposed the frequency domain lifting and Fourier series expansion technique to identify fast varying LTP systems from the free (transient) response of an LTP system. This identification technique has then been applied in [6] to identify NLTI systems in a two-step procedure. Recently, Allen et al. [1] also developed a nonparametric identification scheme from output-only measurements, which gave rise to a novel framework of LTP operational modal analysis based on Wereley's harmonic transfer function (HTF) concept [28,29]. This challenging problem from output-only observations goes beyond the scope of this paper. We instead suppose that the fast varying LTP system under test can be excited with user defined signal (e.g. a multisine signal) such that FRF measurements can be performed. Accordingly, we study comprehensively how the TP distortions behave on FRF measurements. This investigation yields the following major contributions: