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Wavelet-Galerkin approach for power spectrum determination of nonlinear oscillators

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ABSTRACT

A wavelet-Galerkin method based solution for nonlinear differential equation of motion is presented. Specifically, first, theory background of Periodic Generalized Harmonic Wavelet (PGHW) and its connection coefficients are briefly introduced. Next, wavelet coefficients of response are solved from a set of nonlinear algebra equations obtained via the wavelet-Galerkin approach. In this regard, Newton's method is employed to solve the nonlinear algebra equation. Further, stochastic response is determined by evoking a relationship between the wavelet coefficients and the corresponding response power spectrum density. Finally, pertinent numerical simulations demonstrate the reliability of the proposed approach.

1. Introduction

Probabilistic investigation of structural response to stochastic excitation has been widely employed in the stochastic dynamics and reliability analysis of engineering applications. However, it has been argued that the stochastic dynamic analysis of nonlinear systems remains a challenging issue, whereas the linear random vibration theory has appeared in textbooks for almost 60 years. In this regard, contribution by Caughey [1] is among the first reviews to this aspect. Later, stochastic averaging [2,3] and statistical linearization [4,5] attracted researchers' considerable attentions. Generally speaking, one of the reasons for this situation lies in the multi-frequency response of nonlinear systems to mono-frequency excitation. Therefore, the celebrated relationship of power spectrum density (PSD) in the linear random vibration theory does not lend itself to the nonlinear situation in a straightforward manner.

An alternative approach to deal with the problem of multi-frequency transfer function is the Volterra series technique. In this method, response PSD can be calculated by multifold integrals in the frequency domain [6]. Another approach for the determination of PSD was developed by Cai and Lin [7], via the spectrum cumulant-neglect closure technique in the context of polynomial nonlinearities. Relying on the Fourier series expansion of stochastic/sample excitation and stochastic/sample response and the harmonic balance, Bouc and Defilippi [8] developed an approach for the determination of stationary deterministic/stochastic response. Later, based on a similar concept, Spanos, Di Paola and Failla [9] suggested an alternative approach for the response PSD determination with arbitrary nonlinearities, based on a closed form of Fourier transform of cubic and quadratic terms and the cubicization procedure. This approach was further extended to the case of the MDOF system by the same authors [10].

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Note that the concept associated with Fourier transform is stationarity. Therefore, the limitation of the above-mentioned approach of harmonic balance is that only stationary response can be investigated. However, for the seismic excitations of engineering structures, not only the amplitudes of earthquake acceleration, but also the spectrum content varies with time. Therefore, a local basis in time-frequency domain is needed to account for the time-varying spectrum decomposition of the non-stationary processes. In this regard, the newly emerged modern signal processing techniques, such as the short-time Fourier transform [11], the Wigner-Ville distribution [12] and the wavelet transform [13], possess the potential of dealing with such joint time-frequency analysis. In this regard, Basu and Gupta [14,15], Tratskas and Spanos [16] are among the early researchers who attempted to obtain the stochastic response via the time-frequency resolution of wavelet. Recent investigations include a wavelet-based stochastic linearization technique suggested by Spanos and Kougioumtzoglou [17]. However, to the best of the knowledge of the authors, it can be observed that these studies mostly deal with wavelet application in a linear or specific nonlinear oscillator with a few degrees of freedom. Besides, no study on the non-stationary response PSD determination from the perspective of wavelet-based solution of equation of motion has been reported. Therefore, a tentative study of non-stationary deterministic/stochastic response determination from a wavelet-Galerkin perspective, may not only benefit the development of wavelet application in differential equation theory, but also provide an alternative approach for the well-known non-stationary and nonlinear challenge in the random vibration theory.

In this paper, the response PSD of nonlinear oscillators is obtained by the formulation of the wavelet-Galerkin technique. A class of generalized harmonic wavelet (GHW) is first employed as a set of orthogonal bases to solve the nonlinear differential equation of motion. Specifically, first, periodic generalized harmonic wavelets, orthogonal on a finite time interval to each other and locally supported in the frequency domain, are employed to convert the differential equation of motion into a set of nonlinear algebra equations by the wavelet-Galerkin formulation. Next, Newton's iterative method is applied to solve the nonlinear algebra equations and to obtain the unknown wavelet coefficients of nonlinear responses. To this purpose, the Jacobian matrix in Newton's method of the nonlinear algebra equation is divided into a linear and a nonlinear part. Further, closed from of the Jacobian matrix is derived and the Fast Wavelet Transform (FWT) is employed to obtain the wavelet coefficient of nonlinear terms. Finally, stochastic response is therefore obtained by invoking the relationship of squared modulus of wavelet coefficients and the PSD of the response. Pertinent deterministic/stochastic numerical examples including nonlinear stiffness and damping oscillators with different system parameters to uniformly or non-uniformly modulated stochastic excitations demonstrate the promising use of the proposed approach.

2. Theory background

In this section, theory background including GHW and its applications, derivation of periodic GHW and its connection coefficients involved in the Galerkin based solution of the differential equation, as well as wavelet based PSD estimation of stochastic process are introduced.

2.1. Generalized harmonic wavelet

The harmonic wavelet (HW) and the so-called generalized harmonic wavelet, developed by Newland [18,19] in the early 1990s, are kinds of orthogonal wavelets compactly supported in the frequency domain. The bandwidth of each scale of the GHW is controlled by a pair of indices, whereas of the HW is octave banded and determined by a scale index only. This special property renders the wide application of GHW in engineering related applications; take power spectrum estimation [20] for example. Besides, GHW is also employed in the stochastic response determination [16]; see also a detailed review paper by Tratskas and Spanos [21]. Specifically, a wavelet of scale (m, n) and time shift k possesses a representation in the frequency domain as

$$\Psi^{G}_{(m,n),k}(\omega) = \begin{cases} \frac{1}{(n-m)\Delta\omega} e^{-\mathrm{i}(\omega k T_0/n-m)}, & m\Delta\omega \le \omega < n\Delta\omega \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where the superscript "G" denotes "generalized" harmonic wavelet; the subscript (m, n) and k denote the scales and the time translations of wavelets, respectively, with $k = 0, 1, \cdots, N_t - 1$; $N_t = n - m$ being the number of time intervals divided in the time domain; T_0 is the time duration of the signal under consideration and $\Delta\omega = 2\pi/T_0$ is the sampling step in the frequency domain. The time-domain representation of the wavelet can be further yielded by an inverse Fourier transform of Eq. (1), that is

$$\psi^{G}_{(m,n),k}(t) = \frac{\exp[\mathrm{i}n\Delta\omega(t-(kT_0/n-m))] - \exp[\mathrm{i}m\Delta\omega(t-(kT_0/n-m))]}{\mathrm{i}(n-m)\Delta\omega(t-(kT_0/n-m))}. \tag{2}$$

Note that a more compact form of generalized harmonic wavelet has been suggested by Giaralis [22] involving the phase and magnitude of the GHW.