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A robust compressive sensing based technique for reconstruction of sparse radar scenes ☆

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ABSTRACT

Pulse-Doppler radar has been successfully applied to surveillance and tracking of both moving and stationary targets. For efficient processing of radar returns, delay–Doppler plane is discretized and FFT techniques are employed to compute matched filter output on this discrete grid. However, for targets whose delay–Doppler values do not coincide with the computation grid, the detection performance degrades considerably. Especially for detecting strong and closely spaced targets this causes miss detections and false alarms. This phenomena is known as the off-grid problem. Although compressive sensing based techniques provide sparse and high resolution results at sub-Nyquist sampling rates, straightforward application of these techniques is significantly more sensitive to the off-grid problem. Here a novel parameter perturbation based sparse reconstruction technique is proposed for robust delay–Doppler radar processing even under the off-grid case. Although the perturbation idea is general and can be implemented in association with other greedy techniques, presently it is used within an orthogonal matching pursuit (OMP) framework. In the proposed technique, the selected dictionary parameters are perturbed towards directions to decrease the orthogonal residual norm. The obtained results show that accurate and sparse reconstructions can be obtained for off-grid multi target cases. A new performance metric based on Kullback–Leibler Divergence (KLD) is proposed to better characterize the error between actual and reconstructed parameter spaces. Increased performance with lower reconstruction errors are obtained for all the tested performance criteria for the proposed technique compared to conventional OMP and ℓ_1 minimization techniques.

1. Introduction

In many engineering and science applications the objective is to reconstruct an image or a map of the underlying sensed distribution from available set of measurements. Specifically in radar imaging a spatial map of reflectivity is reconstructed from measurements of scattered electric field. State of the art radar systems operate with large bandwidths or high number of channels which generate very large data sets for processing. On the other hand in most of the radar applications the reflectivity scene consists of small number of strong targets. In both cases, significant amount of data is processed mainly to estimate delay and Doppler of relatively few targets. This point raises the applicability of sparse signal processing techniques for radar signal processing.

The emerging field of Compressive Sensing (CS) [1–3] is a recently developed mathematical framework in which the primary interest is to invert or reconstruct a signal $\boldsymbol{x}$ from noisy linear measurements $\boldsymbol{y}$ in the form $\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{n}$. The focus of CS is to solve this linear problem in the underdetermined case where number of measurements is less than the number of unknowns which is very important in decreasing the required amount of data to tolerable levels in radar applications. For a signal $\boldsymbol{x}$ of dimension N that has a K-sparse representation in a transform domain $\boldsymbol{\Psi}$, as $\boldsymbol{x} = \boldsymbol{\Psi s}$ and $\|\boldsymbol{s}\|_0 = K$, CS techniques enable reliable reconstruction of the sparse signal $\boldsymbol{s}$, hence $\boldsymbol{x}$ from $O(K \log N)$ measurements by solving a convex ℓ_1 optimization problem of the following form:

$$\min \|\boldsymbol{s}\|_1, \quad \text{subject to } \|\boldsymbol{y} - \boldsymbol{\Phi\Psi s}\|_2 < \epsilon. \tag{1}$$

CS theory provides strong results which guarantee stable solution of the reconstructed sparse signal for a forward matrix $\boldsymbol{A} = \boldsymbol{\Phi\Psi}$ if it satisfies the restricted isometry property (RIP) [4–6]. It has also been shown that random measurement matrices $\boldsymbol{\Phi}$ with i.i.d. entries guarantees the RIP of $\boldsymbol{A}$ for known basis [7].

Due to these appealing properties of CS and its important advantages for radar, recently CS has received considerable attention in the radar research community. In one of the earliest papers on CS applied to radar, the possibility of sub-Nyquist sampling and

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elimination of match filtering has been discussed [8]. In [9,10], experimental radar imaging results for step frequency and impulse ground penetrating radars have been provided and later extended in [11,12]. To exploit sparsity in the time-frequency domain, high resolution CS-radar has been proposed in [13]. CS based SAR image reconstruction techniques have been proposed in [14]. A CS based MIMO radar has been proposed in [15] for obtaining simultaneous angle and Doppler information. In [16], CS is investigated in distributed radar sensor networks. Further information on the CS based radar applications can be found in [17] and [18].

All the above mentioned sparse reconstruction techniques mainly discretize a continuous parameter space such as range, Doppler or angle and generate a number of grid points where the targets are assumed to be positioned on the nodes of the grid. Under this assumption, the sparsity requirement of CS theory is satisfied and the CS techniques provide satisfactory results. Unfortunately, no matter how fine the grid is, the targets are typically located in off-grid positions. It has been discussed in literature that the off-grid targets creates an important degradation in CS reconstruction performance [19–24]. Off-grid problem is not only observed in CS based radar but many other application areas such as target localization [25], beamforming [26] or shape detection [27], where the sparsity of the signal is in a continuous parameter space and the sparsity basis $\boldsymbol{\Psi}$ is constructed through discretization or griding of this parameter space.

To reduce the sensitivity of the reconstruction to the off-grid targets, denser grids can be used. However, decreasing grid dimensions causes significant increase in the coherence of the compressive sensing dictionary, beyond a certain limit which causes loss of the RIP [7]. To avoid this problem of increased coherence between dictionary columns, in [28], the dictionary is extended to several orthogonal dictionaries and not in a single dictionary, but in a set of them by using a tree structure, assuming that the given signal is sparse in at least one of them. However, this strategy depend on several set of fixed dictionaries generated through discrete parametrization and the main goal is to select the best set of fixed atoms from all dictionaries rather than focusing on basis mismatch. In the works [21–23] the effect of basis mismatch problem on the reconstruction performance of CS is analyzed and the resultant performance degradation levels and analytical ℓ_2 norm error bounds are given. However these works have not offered a systematic approach for sparse reconstruction under parametric perturbations.

There are several approaches in literature for the basis mismatch problem. In Continuous Basis Pursuit approach [29], recovery of sparse translation invariant signals is performed and perturbations are assumed to be continuously shifted features of the functions on which sparse solution is searched for. A dictionary that includes auxiliary interpolation functions that approximates translates of features via adjustment of their parameters is generated and ℓ_1 based minimization is used on primary coefficients. In [24], an algorithm based on the atomic norm minimization is proposed and the solution is found with a semi-definite programming. In [30], ℓ_1 minimization based algorithms are proposed for linear structured perturbations on the sensing matrix where perturbation vectors are modeled as an unknown constant multiplied by a known vector which specifically defines the direction which is typically unknown in practice. Works based on total least square (TLS) as [31,32] assume that general perturbations appear both on the dictionary and measurements. In [31] for solving TLS problem an optimization over all signal $\boldsymbol{x}$, perturbation matrix $\boldsymbol{P}$ and error vector spaces is performed. To reduce complexity, suboptimal optimization techniques have also been proposed. In [32] a constrained total least squares technique is introduced assuming dictionary mismatches are constrained by errors of grid points and a joint estimate of grid point errors and signal support is found by general TLS techniques. In [33], non-parametric perturbations in a bounded perturbation space is considered and some reconstruction guarantees are provided.

This paper mainly focuses on reconstruction of sparse parameter scenes and proposes a novel parameter perturbation based adaptive sparse reconstruction technique to provide robust reconstructions in the off-grid case. The proposed technique is an iterative algorithm that works with a selected set of dictionary vectors that can be obtained via one of sparse greedy techniques such as matching pursuit (MP) [34], orthogonal matching pursuit (OMP) [35], iterative hard/soft thresholding (IHT) [36] or the compressive sampling matching pursuit (CoSaMP) [37]. The parameters of the selected dictionary atoms are iteratively adapted within their grids towards directions that decreases the residual norm. The proposed technique presently is used within the general OMP framework hence named as parameter perturbed OMP (PPOMP). As demonstrated in the reconstruction of sparse delay–Doppler radar scenes, the proposed method is successful in recovering the targets with arbitrary positions. Compared to conventional CS reconstruction techniques like OMP or ℓ_1 minimization, proposed PPOMP technique has achieved lower reconstruction errors for a general delay–Doppler scene in all the conducted performance tests. The general idea of proposed parameter perturbation can also be applied to other areas where discrete parameters are selected from continuous parameter spaces such as frequency or angle of arrival estimation problems.

The organization of the paper is as follows. Section 2 outlines the delay and Doppler data model and formulates the sparse reconstruction problem in CS framework. The proposed parameter perturbation technique and the PPOMP algorithm is detailed in Section 3. Simulation results on variety of examples with performance comparisons are given in Section 4. Section 5 covers conclusions, and direction of possible future work.

2. Delay–Doppler radar imaging: data model and formulation

Coherent radar systems transmit a sequence of pulses with known phases and processes the received echoes to perform clutter suppression and detection at each angle of interest. Excellent references on the operation of radar receivers are available in the literature [38,39]. In this paper we consider a classical pulse Doppler radar with a co-located receiver and a transmitter. Although it is not investigated in here, MIMO radar systems can also be considered within CS framework [15,40]. Let radar transmits $s(t)$, a coherent train of N_p pulses:

$$s(t) = \sum_{i=0}^{N_p-1} p(t - iT_{PRI}) e^{j2\pi f_c t}, \tag{2}$$

where, $p(t)$ is the individual pulse waveform, T_{PRI} is the uniform pulse repetition interval and f_c is the radar carrier frequency. Assuming K dominant targets with delays of τ_{T_m} and Doppler shifts of ν_{T_m}, $1 \leqslant m \leqslant K$, the received signal following the baseband down-conversion can be expressed as:

$$y(t) = \sum_{m=1}^{K} \alpha_m s(t - \tau_{T_m}) e^{j2\pi \nu_{T_m} t} + n(t), \tag{3}$$

where α_m is the complex reflectivity of the individual targets and $n(t)$ is the measurement noise. The above relation between the received signal and target parameters are expressed in terms of the measurable quantities of delay and Doppler. These parameters are related to the range and radial velocity of the mth target as:

$$\tau_{T_m} = \frac{2R_m}{c}, \qquad \nu_{T_m} = \frac{2f_c}{c} v_m, \tag{4}$$