



Wavelet-based Frequency Response Function for time-variant systems—An exploratory study



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ABSTRACT

A wavelet-based Frequency Response Function (FRF) is proposed for vibration analysis of systems with time-varying parameters. The classical FRF is extended to a representation in the combined time–frequency domain using wavelet analysis. Time averaging is performed on the initial FRF to improve signal-to-noise ratio. It is shown that use of the wavelet ridge algorithm is effective in extracting and representing visually data from wavelet-based FRFs. The wavelet-based FRF is demonstrated on selected time-variant simulated lumped parameter systems and one experimental vibrating system.

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1. Introduction

Vibration analysis is important in engineering design and dynamic testing. Vibration analysis is typically performed using eigenvector decomposition in the time domain or frequency analysis which relies on the Fourier or Laplace transforms. The solution of the vibration problem can be viewed also as an input–output relation. The Frequency Response Function (FRF), defined as the frequency-domain ratio between the structural response and the force excitation, is often used to study this relation. The FRF can reveal dynamic properties of structures and is of prime importance in vibration analysis. It is the input–output relationship from which modal parameters (i.e. natural frequencies, damping ratios and mode shapes) can be determined.

Classical vibration analysis assumes that the analysed systems are time-invariant, i.e. the output for such systems does not change with a delay in the input. However, this assumption is not valid for many engineering systems with time-variant coefficients in the corresponding governing equations of motion. Well-known concepts, analytic methods and experimental techniques of linear time-variant analysis cannot be applied to systems with varying physical parameters. Modal analysis has been developed for linear time-invariant systems and is not appropriate for time-variant systems. Conventional definitions of modal parameters are not valid for time-variant systems. Varying mass and/or stiffness leads inevitably to varying natural frequencies and mode shapes whereas system responses to harmonic excitations are non-stationary. Also, the classical FRF for system identification is defined in the frequency domain using the Fourier transform. It is thus unable to accurately represent systems with time-varying properties.

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Various methods have been developed and investigated for time-variant systems. The vast literature on the subject covers different areas of dynamics, control and signal processing. In dynamics the problem is addressed in vibration modelling and analysis. The former often refers to an identification problem. Structural responses are used to construct a model that represents the overall structural dynamics. The latter extracts physical parameters from the model. Various parametric and non-parametric methods for modelling and analysis have been investigated. An excellent review that covers time-variant systems is given in [1].

Vibration analysis of time-variant systems includes methods based on a generalised time-varying modal decomposition similar to [2–10]: the classical Floquet approach, Hilbert transform, empirical mode decomposition joint-time–frequency analysis and time-scale analysis. The last listed approach is particularly attractive due to the time-variant nature of wavelets. There exists a vast literature from the work on damping estimation [11–15] and nonlinear system analysis [16] to recent online identification approaches based on adapted wavelets [17,18]. A good overview of a various wavelet-based approaches can be found in [19–21].

References on output-only analysis of time variant systems include work in the frequency and time domains. The former involves peak picking and frequency domain decomposition. The latter includes the random decrement technique, Ibrahim time-domain method, least square complex exponential method, stochastic subspace identification algorithm or autoregressive moving average methods. A good overview of these techniques and many other approaches is given in [19].

It appears that relatively less work has been done to extend the classical input–output modal analysis to time-variant systems. The majority of these methods are adaptive approaches, developed mainly in control engineering, signal processing and mathematics. The best known approaches are based on time–frequency analysis, as discussed in [1]. Other methods investigated include pseudo-modal parameters based on eigenvalues and discrete-time state transition matrix [22], time-dependent mode shapes based on eigenvectors and Taylor expansion [23], point evolution approach based on complex output signals attached to input signals [24], partial characteristics computed for selected time periods [25], underspread theory [26] or autoregressive moving average models [27]. All above approaches are adaptive transfer functions, mainly for periodic time-variances. A different approach based on wavelet has been proposed in [28], where the transfer function is used for system identification. Within this framework a few attempts have been made to introduce time-variant FRFs. The well known concepts based on evolutionary spectra [29] and the so-called frozen spectra discussed in [1] and based on time–frequency analysis [30–32] are probably the best known approaches. The work in [17] utilises the Littlewood–Paley wavelet and focuses mainly on identification of modal parameters. The method presented in [33] is an evolutionary approach and is also used for modal identification. One of the first attempts to use a wavelet-based Frequency Response Function for vibration analysis can be probably attributed to [20,34], where the continuous wavelet transform was applied to analyse experimental non-stationary vehicle data. The results, however, were inconclusive as the entire concept has not been supported by any reasonable interpretation and the output-by-input wavelet-based division proved to be highly sensitive to noise and singularities.

The aim of the research presented in this paper is to explore a wavelet-based FRF. The focus is on the FRF's amplitude capable of tracking natural frequencies of system vibration as they vary with time. This paper is structured in the following format: for the sake of completeness Section 2 briefly introduces the basic concept of wavelet analysis. Section 3 gives a definition of the wavelet-based FRF and its interpretation. Section 4 gives the interpretation of this function through the convolution for the wavelet transform. Section 5 describes numerical implementation of the wavelet-based FRF. Section 7 presents the application of the method to simulated lumped parameter systems and time-variant experimental data. Section 7 draws conclusions from the research presented and suggests further work on the subject.

2. Wavelet analysis

This section briefly introduces wavelet tools used in the current paper. The continuous wavelet transform is introduced to explain the concept of wavelet ridges required to define the wavelet-based FRF in Section 4.

2.1. Continuous wavelet transform

The continuous wavelet transform can be defined as

$$W_{\psi}(a, b)[x(t)] = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad (1)$$

where b is a translation indicating locality, a is a dilation or scale parameter providing frequency resolution and $\psi(t)$ is an analysing wavelet. The normalisation by $1/\sqrt{a}$ in the above equation ensures that the integral energy given by each translated and dilated wavelet $\psi_{a,b}(t)$ is independent of the dilation a . It is important that analysing wavelets satisfy the admissibility condition given by

$$\int_{-\infty}^{\infty} \frac{\psi(t)^2}{|f|} df < \infty \quad (2)$$

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