Mechanical Systems and Signal Processing 47 (2014) 66–77

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Hilbert transform methods for nonparametric identification of nonlinear time varying vibration systems

Michael Feldman *

Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Technion City, 32000 Haifa, Israel

ARTICLE INFO

Article history:
Received 2 January 2012
Received in revised form
31 August 2012
Accepted 2 September 2012
Available online 26 September 2012

Keywords:
Non-linear time-varying system
Hilbert transform
Nonparametric identification
Envelope
Instantaneous frequency

ABSTRACT

The objective of this paper is to demonstrate a Hilbert transform (HT) method for identification of mechanical time-varying vibration systems under free and forced vibration regimes. This special kind of non-stationarity arises in experimental vibration analysis and in engineering practice. The method is based on the HT of input/output signals in a time domain to extract instantaneous dynamic structure characteristics, such as natural frequencies, stiffness, damping, and their variations in time.

The HT assigns a complementary imaginary part to a given real signal part, or vice versa, by shifting each component of the signal by a quarter of a period. Thus, the HT pair provides a method for determining the instantaneous amplitude and the instantaneous frequency of a signal.

For general non-stationary vibration signals, the analytic signal method does a good job of simultaneous time-frequency localization of the main signal components.

The paper focuses on HT signal processing techniques and identifies three groups of dynamics time-varying SDOF systems: slow varying quasi-periodic modulation of stiffness under free and forced vibration, slow varying quasi-periodic modulation of nonlinear stiffness under free vibration, and fast inter-wave parametric stiffness modulation (Mathieu equation).

1. Introduction

Typically, a vibration system concerned with a mechanical construction having mass, stiffness and damping elements is a physical structure that, when taking an impulse or another input force, usually initiates an output natural vibration motion. If the system has a time-varying mass or stiffness parameter, the natural frequency will depend decisively on the variation. If the system also has a time-varying damping, the solution amplitude will react in situ with the damping variation.

Such frequency and amplitude variations can be clearly detected by any suitable signal processing technique in the time or frequency domain. The appropriate signal processing is mostly based on the extracted features of the level and frequency content of a varying signal in the form of the envelope and the instantaneous frequency. Thus, for example, a moving least-squares technique and a Kalman filter allow time-varying frequency tracking, while the traditional Fourier transform and wavelets technique can identify online variation in the natural frequencies and mode shapes of a vibration system [1–3].

* Tel.: +972 4829 2106; fax: +972 4829 5711.
E-mail address: MFeldman@technion.ac.il

0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2012.09.003

If identification of the varying dynamic system requires additional a priori information about the dynamic system and its vibration model, this identification is called parametric, and its aim is to determine the parameters of a known model. A dynamic system model is commonly created on the basis of knowledge of a mechanical construction and a functional operation of a technical object [4]. In some other more sophisticated cases, a nonparametric analysis may be required to estimate the parameter variation of the unknown physical equation of the dynamic system [5,6]. Nonparametric system identification of the modal parameters, such as frequencies, damping, and mode shapes as functions of the physical inertia, stiffness, and damping properties, is based only on analysis of the system behavior.

Recent works in the area of time domain representations of vibration based on the HT show great promise for applications in nonparametric vibration system identification. An outstanding work by N. Huang, known as Empirical Mode Decomposition, adaptively decomposes a signal into the simplest intrinsic oscillatory modes (components) in the first stage. Then, in the second stage, each decomposed component forms a corresponding set of instantaneous amplitudes and frequencies called the Hilbert spectrum that can be used to obtain time-frequency vibration representation (characterization). Every significant decomposed elementary component holds exact amplitude and frequency information contained in the original signal, and the HT method makes it sensitive to any modal shift.

Practically all publications dedicated to HT time domain nonlinear vibration identification with envelope and instantaneous frequency use the approach suggested in [7]. This FREEVIB approach introduces a vibration solution as a new complex variable which, when substituted into an initial differential equation, directly determines the system's natural frequency and damping parameters. The proposed methods for identifying instantaneous modal parameters (natural frequencies, damping characteristics and their dependencies on a vibration amplitude and frequency) prove to be very simple and effective. The HT-based signal processing in parallel with other time-frequency representations in the form of Wavelet transform, Gabor Transform, or Wigner–Ville distribution provides a powerful tool for vibration analysis and identification of vibration systems [8].

Of particular interest and importance is the use of the HT to interpret nonlinear system motion. The measured instantaneous characteristics, such as the instantaneous frequency or the amplitude of the nonlinear solution, take an unusual fast oscillation (intrawave modulation) form. In the case of supplementary parameter variation, both instantaneous functions of each vibration mode – the envelope and the instantaneous frequency – become complicated modulated functions: they reflect the slow time variation of the stiffness plus fast intrawave modulations due to nonlinearity [8]. Therefore, only a smeared or blurred form of the instantaneous frequency will be obtained through direct frequency monitoring.

The main purpose of this paper is to demonstrate the method of nonparametric identification of a vibration system when the linear time-varying system description fails because the non-linearity is too severe and a very accurate model is needed. In this regard, not only must the presence of a parameter variation and nonlinearity be detected, but also an adequate and readily interpretable system model must be identified. It has been suggested that HT processing involving Hilbert vibration decomposition and congruent functions can identify the time-frequency distribution of a signal with great accuracy.

The digital HT signal processing the same as every other approach has some inherent limitations. The method is based on the multipoint FIR digital Hilbert transformer and therefore it requires significantly longer initial data to neglect the filter end effects. The method has a good but an ultimate frequency resolution required for separation of closely spaced harmonics. It allows separation the finite small number of valued components, which does not exceed 5–6 components. It is very sensitive to measured instrumental noise. The HT decomposition method is not effective for the separation of non-oscillation types of motion, such as random, impulse, slow aperiodic signals.

2. Hilbert transform-based identification and analysis of time varying vibration systems

2.1. The Hilbert transform in time domain approach.

The HT assigns a complementary imaginary part $\tilde{x}$ to a given real signal part x, or vice versa, by shifting each component of the signal by a quarter of period $\tilde{x} = H[x]$. Thus, the HT pair provides a method for determining the instantaneous amplitude $A(t) = \pm\sqrt{x^2(t) + \tilde{x}^2(t)}$ and the instantaneous frequency of a signal $\omega(t) = d[\arctan(\tilde{x}(t)/x(t))]/dt$.

For a general non-stationary vibration signal, the analytic signal method does a better job of simultaneous time-frequency localizing of the main signal components [8]. The analytic signal method is equally applicable to deterministic and random processes, although, generally speaking, it does not divide them into two separate groups. Hence, it enables us to investigate any oscillating time function from a more general point of view. The method is also good for solving problems concerning analysis of stationary and non-stationary vibrations, as well as narrow- and/or wideband multi-component signals. It also allows precise analysis of the transformation and dissipation of vibration energy, and vibration effects on machine durability.

2.2. Advantages of Hilbert transform time domain identification

Taking analytic signal representation into account enables one to consider a vibration process, at any moment in time, as a quasi-harmonic oscillation, amplitude- and frequency-modulated by time-varying functions $A(t)$ and