Mechanical Systems and Signal Processing 47 (2014) 94–119

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Parametric modal identification of time-varying structures and the validation approach of modal parameters

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ARTICLE INFO

Article history:
Received 31 January 2012
Received in revised form
12 March 2013
Accepted 31 July 2013
Available online 16 September 2013

Keywords:
Modal parameter estimation
Time-varying structure
Two-stage least square
Fuzzy clustering

ABSTRACT

This paper investigates the problem of output-only modal parameter estimation of time-varying structures. A two-stage least square method for modal parameter estimation of time-varying structures is presented, consisting of a least square complex frequency-domain stage for the modal parameter estimation of all discrete time samples and a weighted least square stage for the estimation of continuous-time-represented modal parameters. This paper introduces a fuzzy-clustering-based approach for validating and sifting the modal parameters and develops four modal assurance criterion-based distance functions to improve the quality of clustering of operational mode shapes. Furthermore, the proposed method has been validated by a simulation example and a laboratory experiment.

1. Introduction

In this paper, we consider linear slowly time-varying structures [1,2]. Time-varying (TV) structures are generally characterized by time-dependent structural dynamic parameters, such as stiffness, damping ratio and mass, which may change with time. In reality, many engineering structures, such as traffic-excited bridges [3], launch vehicles with varying fuel mass [4], airplanes in flight with varying additional aerodynamic effects [5], deployable and flexible geometry-variable aerospace structures, rotating machinery etc., show properties changing with time. On one hand, the changes of all these structures are obvious and unavoidable in terms of their corresponding structural dynamic analyses and applications. On the other hand, the timescales of the changes are different, such as tens of seconds for traffic-excited bridges, hundreds of seconds for space launch vehicles, couples of hours for civil structures such as bridges or football stadiums [6]. Furthermore, in many real-life applications, the excitation on the time-varying structures is unknown and random so that operational or output-only methods are appropriate.

The currently existing methods of modal parameter estimation for time-varying structures can be classified into two categories: parametric time-domain methods and non-parametric time–frequency-domain methods.

In the past decades, many time-domain parametric approaches of dynamic identification for time-varying structures were presented. There are two categories of time-domain parametric identification approaches for time-varying structures: autoregressive moving average model based and state-space model based approaches. In the first category, Petsounis and Fassois [7,8] presented the time-dependent autoregressive moving average (TARMA) representation for modeling

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0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2013.07.021

non-stationary stochastic vibration. Poulimenos and Fassois [2] surveyed and compared several approaches of TARMA-based non-stationary random vibration modeling including unstructured parameter evolution, stochastic parameter evolution, and deterministic parameter evolution. Spiridonakos et al. [9,10] estimated the modal parameters via the TARMA-based approaches and validated them with a laboratory experiment. In the second category, Liu [11,12] proposed a state space based approach for linear time-varying systems via decomposing a series of Hankel matrices that are assembled by output response data or additional input data with singular value decomposition (SVD). Liu and Deng [13,14] improved the state space based approach for the linear time-varying system by making it less sensitive to noise and validated the identification algorithm using an experiment of a moving cantilever beam.

Time–frequency analysis is very popular in signal processing, especially for nonstationary stochastic signals. The major advantage of time–frequency analysis is that it describes the spectral variation in function of time [15,16]. Several non-parametric, also called unstructured, time–frequency analysis-based time–frequency-domain identification approaches were developed in the past decade. Ghanem and Francesco [17] presented a wavelet-based identification approach, which transforms the classic governing equation of motion into a wavelet expanded form by projecting the physical responses to a series of wavelet coefficients and identified the modal parameters by solving the expanded-form equation. Roshan-Ghias et al. [18] estimated modal parameters using the smoothed pseudo Wigner–Ville distribution (SPWVD), which represents the free responses onto WVD plots using the analytical expressions of the free responses and estimates the natural frequency and damping ratio of the systems by tracking the ridges of the WVD plots. Xu [19] presented an approach of modal parameter estimation through Gabor expansion of response signals. Meanwhile, some approaches using Hilbert transform (HT) or Hilbert–Huang transform (HHT) were proposed. Spina et al. [20] used the Gabor expansion to identify the modal matrix, which decouples the transient response into a set of uncoupled quasi-harmonic components, and used the HT to analyze these quasi-harmonic components further. Xu et al. [21] decomposed the responses into a series of single components with Gabor expansion and identified the modal parameters of these single-component signal with HT. Shi et al. [22] decomposed the responses with empirical mode decomposition (EMD).

Different from the aforementioned parametric time-domain methods or non-parametric time–frequency-domain methods, this paper presents a parametric time–frequency-domain method of modal parameter estimation for time-varying structures based on the parametric frequency-domain method. For linear time-invariant (LTI) structures, many frequency-domain estimators for modal parameters were studied using input–output or output-only data sets in the past decades. Most frequency-domain estimators employ parametric models [23] including common-denominator models (CDM), left or right matrix fraction descriptions (LMFD or RMFD), partial fraction models (PFM) and state space models (SS). On the other hand, there are many kinds of MDOF output-only estimators in the frequency-domain. Guillaume et al. [24] presented a maximum likelihood estimator based on the common-denominator model for experimental and operational modal parameter estimation. Brincker et al. [25] presented the frequency-domain decomposition (FDD) to estimate modal parameters by decomposing the power spectral density (PSD) of responses directly. Guillaume et al. [26] and Van der Auweraer et al. [27] developed the least square complex frequency-domain estimator based on the common-denominator model and linear least squares. Guillaume et al. [28] presented the polyreference LSCF method (PolyMAX). Van Overschee and De Moor [29] presented a frequency-domain subspace estimator based on the state space model and singular value decomposition. Cauberghe [30] overviewed some frequency-domain estimators based on the common-denominator model.

This paper attempts to estimate modal parameters of time-varying structures using a two-stage least square method. There are four phases in this method: (1) the non-parametric estimation of time-dependent PSD using time–frequency analysis, (2) the estimation of the PSDs and the modal parameters at each time sample using the least square complex frequency-domain method, (3) a fuzzy-clustering-based approach of modal parameter validation, and (4) the estimation of the continuous-time function of the modal parameters using a weighted linear least square approach. The second and fourth phases are the two stages of the least square estimation in the proposed method.

This paper is organized as follows: time–frequency analysis and fuzzy cluster analysis are briefly introduced in Sections 2 and 3. The mathematical modeling of the two-stage least square estimator is proposed in Section 4. Sections 5 and 6 present a simulation example and an experimental example to validate the proposed method respectively. Finally, some conclusions are drawn in Section 7.

2. Time–frequency analysis

2.1. Time–frequency analysis and time-dependent power spectral density

In the conventional frequency-domain methods of modal parameter estimation, the frequency response function (FRF) or the PSD is estimated by some non-parametrical approaches, such as periodogram and correlogram, before the estimation of modal parameters. However, these approaches cannot correctly obtain the time-dependent PSDs of non-stationary responses in case of time-varying structures. In this paper, the time-dependent PSDs are non-parametrically estimated by time–frequency analysis before the two-stage least square estimation of modal parameters.

Time–frequency analysis can be classified as follows. The first category represents the signals by time–frequency functions by translation, modulation and scaling, such as short-time Fourier transform (STFT) and wavelet transform (WT). The second category is the bilinear time–frequency distribution (TFD), commonly called Cohen's class, which represents the behavior of the energy distribution of the signals, i.e. the concentration of energy at certain time and frequency regions [31].