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Modal parameter estimation using interacting Kalman filter

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ABSTRACT

The focus of this paper is Bayesian modal parameter recursive estimation based on an interacting Kalman filter algorithm with decoupled distributions for frequency and damping. Interacting Kalman filter is a combination of two widely used Bayesian estimation methods: the particle filter and the Kalman filter. Some sensitivity analysis techniques are also proposed in order to deduce a recursive estimate of modal parameters from the estimates of the damping/stiffness coefficients.

1. Introduction

Usual system identification techniques for civil and mechanical structures assume the availability of large set of samples derived from a stationary quasi steady structure. On the opposite, several scenarios involve time varying structures. For example, due to interaction with aerodynamics in aeronautics, some critical parameter may have to be monitored, for unstability monitoring (leading possibly to flutter) of in flight data due to fuel consumption and speed change. This relates to the monitoring of time varying structural parameters such as frequencies and damping ratios.

Dynamic systems parameters estimation, and more generally system identification, is an ongoing active topic of research. A wide variety of methods are used, such as stationary or recursive methods like subspace approaches based on state-space models [1–5] and adaptive methods like maximum likelihood or least square [6–11] among them methods based on Auto-Regressive Moving Average (ARMAX) models [12–14] have been widely investigated to estimate flutter indicators.

The main difficulty in estimating the system parameters in a state space model is that in addition to the parameters, state variables are also unknown and unmeasured. Therefore it is important to use an approach that combines state and parameter estimation. It has been shown in [15] that extended Kalman filter based approach gives in general biased or divergent estimate. Moreover, in [16], a comparison between particle filter and EKF based on several examples shows that the particle filter gives a better parameter estimation, especially when non-linearity dealing with parameter estimation is not neglectable. Thus, the particle filter is a good candidate for parameter estimation.

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A typical particle filter algorithm consists in considering the state as a random variable following a Markov process and approximating it by a cloud (set) of random particles mimicking its evolution according to the process equation. Each particle represents a possible state value, and at a given time it has a weight corresponding to its probability of being the most likely state value according to the measurement equation. At a given time, the estimated state is the weighted sum of all particles.

The particle filter yields to a good parameter estimation as shown in [16], but the drawbacks of considering the parameters as auxiliary state variable are first that the computation of the weight of a particle representing an augmented state is not accurate enough because of the nested dependencies between the state and the parameters, second that the number of particles needed to have an acceptable approximation explodes with the dimension of the state which makes the standard particle filter inefficient for high dimensional systems. A solution is to modify particle filter algorithm in order to take advantage of the fact that for linear systems, the state can be analytically marginalized out conditional on the parameter according to the system equations. Such filters have been introduced in [17,18] as a Rao–Blackwellised particle filter and in [19] as an interacting Kalman filter. From the pure mathematical point of view, interacting Kalman filters can be encapsulated into particle filter models associated with Kalman predictor signals and virtual observation models. The first well founded and rigorous performance analysis of particle filters can be found in [20].

The main idea of an interacting Kalman filter is to consider particles evolving in the parameter space. For each particle, a corresponding state is estimated by applying a Kalman filter to the system in which parameter values are replaced by values associated to this particle. The weight of each particle is computed from the likelihood of the parameter sample it represents and its corresponding state. This results in a bank of adaptive Kalman filters combined with a particle filter.

However, when the system has a relatively big number of parameters which are significantly varying in time, the interacting Kalman filter algorithm still need to be improved. In fact, the accuracy of stochastic simulation-based methods depends essentially on the accuracy of the computation of particle weight, thus it is important to make the best use of the available measurements to compute these weights. Since a particle has as many components as the number of parameters it represents (dampings, stiffnesses), it is wise to take into account how each component of a particle influences the computation of its weight. Accordingly, a contribution of this paper is to show that decoupling stiffness and damping probability distributions when considering the parameters as a set of random variables and having two different Kalman based parameter estimation algorithms running in a combined way result in better parameter estimates overall.

Another contribution of this paper is to introduce a method to recursively track the variations of the modal parameters instead of the damping and stiffness coefficients as it is conventionally done [16,21,18] and which is at first sight easier to handle because tracking directly the modal parameters needs the use of the representation of the system in the modal basis and this involves explosion in the parameter dimension incompatible with a proper implementation of particle filter estimation. The idea is to integrate some sensitivity analysis techniques [22] to the main algorithm in order to track modal parameters with no additional computational cost. In the first section, the structural health monitoring problem is presented. In the second section a full description of the main algorithm is given. Finally, numerical simulations based on the example drawn in [16] are given.

2. Dynamical model and structural parameters

Consider a dynamical system of dimension n whose behavior is given by the following equation:

$$M_t\ddot{x}(t)+C_t\dot{x}(t)+K_tx(t)=\sigma u(t), \tag{1}$$

where M_t, C_t and K_t are respectively the time varying matrices of mass, damping and stiffness, $\sigma \in \mathbb{R}$ and u is the input force. u is modeled as a white Gaussian noise with time varying covariance matrix R^u. Dynamics of M_t, C_t and K_t is assumed to be unknown. Denoted by

$$X(t)=\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad A_t=\begin{bmatrix} 0 & \mathcal{I} \\ -M_t^{-1}C_t & -M_t^{-1}K_t \end{bmatrix} \quad \text{and} \quad \Sigma_t=\begin{bmatrix} 0 \\ M_t^{-1}\sigma \end{bmatrix} \tag{2}$$

Eq. (1) is equivalent to

$$\frac{\partial X(t)}{\partial t}=A_tX(t)+\Sigma_tu(t) \tag{3}$$

Denoted by $X_k=X(k\delta)$, $M_k=M_{k\delta}$, $C_k=C_{k\delta}$, $K_k=K_{k\delta}$, $A_k=A_{k\delta}$, $u_k=u(k\delta)$ and $F_k=e^{\delta A_k}$. Suppose that for $t \in [k\delta, (k+1)\delta[$, $A_t{=}A_k$ and $\Sigma_t=\Sigma_k$ and consider a time discretization at a rate δ. The discretized linear state-space model is

$$X_{k+1}=F_kX_k+\left(\int_0^\delta \exp(A_k v)\Sigma_k\, dv\right) \quad u_k := F_kX_k+B_ku_k. \tag{4}$$

The measurement equation is

$$Y_k := D_kX_k+H_kv_k.$$

Now describe the structural characteristics of the system (1). As the poles of a time varying system are not defined, it is supposed that for each t the time-variation of the system is freezed in order to obtain for each t a different time-invariant