Mechanical Systems and Signal Processing 47 (2014) 151–174

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Frequency domain, parametric estimation of the evolution of the time-varying dynamics of periodically time-varying systems from noisy input–output observations

E. Louarroudi [a,*], J. Lataire [a], R. Pintelon [a], P. Janssens [b], J. Swevers [b]

[a] Vrije Universiteit Brussel, Department of ELEC, Pleinlaan 2, 1050 Brussels, Belgium
[b] Katholieke Universiteit Leuven, Department of PMA, Celestijnenlaan 300 B, B-3001 Leuven, Belgium

ARTICLE INFO

Article history:
Received 1 February 2012
Received in revised form
10 November 2012
Accepted 11 March 2013
Available online 25 May 2013

Keywords:
Linear time-periodic
Frequency domain parametric identification
Errors-in-variables
Harmonic transfer functions
Frozen transfer function
Multisine excitations

ABSTRACT

This paper presents a frequency domain, parametric identification method for continuous- and discrete-time, slow linear time-periodic (LTP) systems from input–output measurements. In this framework, the output as well as the input is allowed to be corrupted by stationary noise (i.e. an errors-in-variables approach is adopted). It is assumed that the system under consideration can be excited by a broad-band periodic signal with a user-defined amplitude spectrum (i.e. a multisine), and that the periodicity of the excitation signal T_{exc} can be synchronized with the periodicity of the time-variation T_{sys} (i.e. $T_{exc}/T_{sys} \in \mathbb{Q}$), such that the system reaches a steady state (a periodic solution). T_{sys} is also known as the *pumping* period. Once the parametric estimation of the time-evolution of the system parameters has been performed, the system model is evaluated at the level of the *instantaneous* transfer function (also known as *system* function, or *parametric* transfer function), which rigorously characterizes LTP systems. If the dynamics of the LTP system are *slowly* varying or the system is linear parameter varying (LPV), a *frozen* transfer function approach is provided to easily visualize and assess the quality of the estimated model. To give the estimated quantities a quality label, uncertainty bounds on the model-related quantities (such as the time-periodic (TP) system parameters, the frozen transfer function, the frozen resonance frequency, etc.) are derived in this paper as well. Besides, a clear distinction between the *instantaneous* and the *frozen* transfer function concept is made, and both can be estimated with the proposed identification scheme. The user decides which transfer function definition suits best its purpose in practice. Finally, the identification algorithm is applied to a simulation example and to real measurements on an extendible robot arm.

1. Introduction

The linear time-invariant (LTI) identification techniques are well-established nowadays [1,2]. However, there are situations where the time-invariant (TI) assumption is not met. This happens in real-life systems where the dynamics are changing over time [3,4]. The causes of the time-variation in the system can be categorized as follows:

- The time-varying character is inherently present in the system and is dictated by the random nature of the system [5–9] (e.g. pitting corrosion in metals, vibration of a helicopter rotor, (bio-) chemical processes, changes of the bio-impedance in the heart, etc.).

* Corresponding author. Tel.: +32 2 629 28 44; fax: +32 2 629 28 50.
E-mail address: elouarro@vub.ac.be (E. Louarroudi).

0888-3270/$ - see front matter
http://dx.doi.org/10.1016/j.ymssp.2013.03.013

- The time-varying behavior is artificially created by an external parameter, the so-called scheduling parameter [10–12] (e.g. extendible robot arm, flight flutter, electronic circuit whose properties depend on an external voltage source, etc.). This kind of system fits into the framework of linear parameter varying (LPV) systems, where the dynamics of the system depend on the external (scheduling) parameter. When the scheduling parameter varies over time, the systems' dynamic behaviors become time-varying.
- When a nonlinear time-invariant (NLTI) system is linearized around the periodic (stable) orbit of the nonlinear system, the linearized system will exhibit a time-varying behavior in the neighborhood of the stable orbit [13,14] (e.g. mechanical systems with a nonlinear stiffness, power distribution networks, etc.).

Many physical structures, in particular mechanical structures, are described by partial differential equations (structural analysis, modal analysis, heat transfer, fluid mechanics, electromagnetism, etc.), where it is a common practice to use finite element methods (FEMs) to solve the problem. The FEM gives rise to a set of ordinary differential equations (ODEs), where each matrix element determines the physical relationship between the applied inputs at some points and the resulting outputs at others. Therefore in a given frequency band, it can be modeled by an ODE at a certain point in the structure (described as a transfer function of a Single-Input Single Output (SISO) system). This framework of lumped systems simplifies the analysis of the system under consideration and the corresponding identification methods. Extending this idea to systems (structures) with a time-varying behavior shows that it is meaningful to describe time-varying systems in a given frequency band with time-varying ODEs.

There exists a variety of model structures in the literature to describe linear time-periodic (LTP) systems [4]. Three model representations, either in continuous or discrete time, are commonly used in practice. First, the (harmonic) state-space approach, that gives rise to the harmonic transfer function (HTF) matrix operator in the frequency domain, which is closely related to the Floquet theory, is described thoroughly in [15,16]. The HTF operator is established by either the harmonic balance method or by making use of convolutions. Furthermore, the input–output formulation via the concept of time-varying impulse response or kernel has also been utilized for the identification either in the time [17,18] or in the frequency domain [19]. Finally, another elegant description for LTP systems is an ordinary differential (difference) equation (ODE) with time-periodic (TP) parameters [20,21].

In this article the focus is put on *weakly* nonlinear time-periodic (NLTP) systems, where it is assumed that the weakly NLTP system can be well-described by a SISO ODE with TP parameters. From the simple ODE model it is then possible to readily derive a *frozen* transfer function (FTF) concept by freezing the ODE parameters in time. However, in the literature the HTF concept is often used [8,15,16,22,23], which is closely related with the Floquet theory for LTP systems [15,16]. This brings us to the first major contribution of the paper, namely:

- summarizing the system theory: a detailed discussion of the distinction between the FTF and the HTF concepts, and establishing the relation with the Floquet theory;
- applying these FTF and HTF concepts on numerical and real measurement examples.

(N)LTP systems have been covering a wide range of applications, especially in the field of mechanics where many systems that sustain a periodic motion (e.g. gear boxes, electrical motors, fans, shafts, helicopter blades, etc.) show a TP behavior. This is, for instance, the case in a twisted-actuated helicopter rotor blade, or in the blades of wind turbines where the periodic time-variations show up due to the periodic change of the aerodynamic properties of the air [6,22]. A nice overview of cyclostationary processes, illustrated with many real-life examples, is given in [7]. More examples can be found in other engineering applications like in control, sampled data systems, multi-rate filter banks, power distribution networks, etc. [4,14,24,25].

From an identification point of view, all the LTP models described above can be seen as belonging to the same class, as long as the base frequency of the time variation $f_{\text{sys}} = 1/T_{\text{sys}}$ (i.e. the pumping frequency) is either known, or can be estimated from the data. Numerous parametric identification methods for LTP systems in different fields of engineering have been described in the literature as summarized in the following. Different parametric (discrete-time) time-domain methods exist for identifying arbitrary and periodically time-varying dynamics of structures/systems, on the one hand, from experiments with an exogenous input [26], and on the other hand, from output-only data [27,28]. These methods are applied mostly on structures/systems with slow dynamic variations (such as moving mass distributions), such that concepts of a *frozen* structure/system and of *frozen* modal parameters make sense in practice [26–29]. Using the lifting and the Fourier series expansion technique, [23] presented a frequency domain Multi-Input Multi-Output (MIMO) LTI method to identify continuous-time state-space LTP models from the free response of mechanical systems. This identification technique is applied in [13] to identify NLTI systems in a two-step procedure. Additionally, in [22] a frequency domain identification scheme is developed to identify the behavior of the blades of wind turbines from output-only measurements. The Floquet modal parameters (i.e. Floquet exponents and the corresponding mode shapes) are extracted from the output power spectrum using classical LTI techniques (e.g. the peak-picking algorithm). Recently, an identification method in [30], using a wavelet-based state space approach, is elaborated to estimate the time-varying system matrix $\mathbf{A}(t)$ and the corresponding *frozen* modal parameters (i.e. eigenvalues of $\mathbf{A}(t)$). In the digital processing world, discrete-time LTP systems are more suited [24,25], where [31–33] proposed a frequency domain estimate of the alias components of LTP systems.