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Benefits of averaging lateration estimates obtained using overlapped subgroups of sensor data ☆

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ABSTRACT

In this paper, we suggest averaging lateration estimates obtained using overlapped subgroups of distance measurements as opposed to obtaining a single lateration estimate from all of the measurements directly if a redundant number of measurements are available. Least squares based closed form equations are used in the lateration. In the case of Gaussian measurement noise the performances are similar in general and for some subgroup sizes marginal gains are attained. Averaging laterations method becomes especially beneficial if the lateration estimates are classified as useful or not in the presence of outlier measurements whose distributions are modeled by a mixture of Gaussians (MOG) pdf. A new modified trimmed mean robust averager helps to regain the performance loss caused by the outliers. If the measurement noise is Gaussian, large subgroup sizes are preferable. On the contrary, in robust averaging small subgroup sizes are more effective for eliminating measurements highly contaminated with MOG noise. The effect of high-variance noise was almost totally eliminated when robust averaging of estimates is applied to QR decomposition based location estimator. The performance of this estimator is just 1 cm worse in root mean square error compared to the Cramér–Rao lower bound (CRLB) on the variance both for Gaussian and MOG noise cases. Theoretical CRLBs in the case of MOG noise are derived both for time of arrival and time difference of arrival measurement data.

1. Introduction

Localization is an important problem encountered in a diverse area of applications such as finding the employees or utilities in a working area, location aware services supplied to mobile phone users, bioinstrumentation and communicating toys [1,2]. A popular method of localization is lateration where measured distances from sensors to the point to be localized are used [3]. Some of the lateration based methods require solving a set of nonlinear equations what is usually performed by some iterative procedures as in the case of nonlinear least squares type algorithms in [4–6]. On the other hand, there are some other lateration based methods which reduce the original nonlinear problem into a linear one making a closed form solution easily obtainable such as the least squares-time difference of arrival (LS-TDOA) [7], least squares-time of arrival (LS-TOA) [8,9] and the so-called ordinary least squares-time of arrival (OLS-TOA) [5] location estimators. Another closed form location estimator is based on Cayley–Menger determinants and uses the geometric properties of tetrahedrons whose corners are defined by the unknown location and three known locations [10]. Some of the closed form location estimators reduce the nonlinear problem partially to a linear system of equations whose solution is obtained in terms of another unknown which is found solving a quadratic equation. One of the resulting two solutions of this quadratic equation is easily eliminated and substituting the solution into the linear system of equations the location is estimated. The time of arrival (TOA) based location estimator in [11], its QR decomposition based version in [4] and another one able to utilize more than necessary distance measurements [12,13] which is in contrast with the former two estimators, belong to this class of estimators. Although these methods have some performance inferiority with respect to the iterative nonlinear methods, their computational simplicity make them preferable in some applications where limited computational power is available.

One type of location estimation scenarios may supply more than necessary distance measurements. In that case either all of the measurements can be used at once to obtain a single location estimate or subgroups of measurements can be selected to obtain many estimates. In the latter, the final location estimate is found as the average of the estimates. We will call this approach as *averaging laterations* while the former one will be called as *single lateration*. The Divide-And-Conquer (DAC) approach of [14] uses averaging laterations with subgroups of distance measurements which might overlap or not. Non-overlapping subgroups necessitate a large number of measurements which property is not shared by the choice of overlapping subgroups [8,15]. In [8], TOA-based

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averaging laterations method with overlapping subgroups of size three was given as an alternative to LS algorithm in finding an unknown location in two dimensions. Recently, in [15], the QR decomposition based TOA method of [4] was applied in regular or robust versions of averaging with overlapping subgroups of measurements. In fact, for fairness in overlapping subgroups, every possible combination of total measurements with the chosen group size is considered in [15] which is also the adopted method in this work. On the other hand, every lateration-based location estimator utilizing LS algorithm can operate in *single lateration* mode with any redundant number of measurements. However, for estimators which cannot handle excessive number of measurements which is more than their nominal number such as the estimators of [11,4], the adopted *averaging laterations* mode of operation seems to be the best candidate in utilizing all of the available measurements while assuring fairness among them.

In order to compare *averaging* and *single* lateration modes of operation, a location estimator which can utilize both nominal or more than nominal number of measurements is required. We name this property as the *scalability* of the estimator. A good example for non-scalable location estimators is the QR decomposition based lateration technique in [4] which always uses three distance measurements. Many of the location estimators having closed form solutions are scalable such as LS solutions of TOA and TDOA type problem formulations. In general, iterative location estimators mainly solving nonlinear equations are scalable too. However, closed form solution producing methods which transform the nonlinear equation systems partially to a linear system of equations, are not scalable with the exception of the methods in [12,13]. Even though iterative methods are scalable, they are not considered for averaging laterations since they are both computationally demanding already and their optimality will be disturbed unless whole data is considered. So, in the first part of our study we chose two TOA based and one time difference of arrival (TDOA) based least squares (LS) estimators for investigating averaging estimates.

Simulation experiments in our study show that generally averaging the estimates obtained with partial sensor data achieves similar performance compared to the case of using whole data in obtaining a single estimate when the measurements contain Gaussian disturbances. However, when some of the measurements are very noisy, the performance of a single lateration based estimator deteriorates significantly. Such measurements which are substantially different from the other measurements are many times called as *outliers*. The disturbing effect of outlier measurements can be eliminated by a robust location estimator [16,17]. Robust location estimators can be classified into two groups: outlier detection based or robust estimation based [17]. Outlier detection based methods eliminate detected outliers completely whereas robust estimation methods lessen their weights in the estimation. For a detailed comparison of these methods, one can refer to [17]. As a robust estimation example, recently in [18], the least median of squared errors obtained in a TDOA based LS solution is minimized over the set of every possible subgroup combination of measurements. An outlier elimination procedure is applied in [19] in order to eliminate outlier estimates obtained with minimal subgroups of measurements and with an iterative nonlinear LS solution based on first order Taylor series expansion of nonlinear localization equation. Then the final estimate is obtained as the median of the qualified estimates. Still some other robust estimation solutions exist such as the genetic algorithm based TDOA solution in [19] and the location estimate in [20] obtained by utilizing the expectation maximization algorithm for removing outlier distance measurements iteratively.

The main idea of this paper is to promote *averaging laterations* as opposed to *single lateration*. Additionally, it is demonstrated that the averager can be easily transformed into a robust version which can handle outlier measurements. The averaging laterations approach in this paper can be considered as an extension of the work in [8] of subgroups with three measurements for TOA based lateration in two dimensions to any possible subgroup size of measurements for lateration in three dimensions. For the localization scenario with outlier measurements, the proposed robust method can be classified as an outlier detection based location estimator like one of the methods in [19] and the new robust averager used for detecting outliers resembles to the modified trimmed mean (MTM) averager defined in [21] which will be described in Section 4. However in this work varying subgroup sizes for measurements are investigated which was not considered before. The outlier statistics was a general mixture model in [19]. Here the statistics of measurement noise with outliers is modeled by a mixture of Gaussians (MOG) distribution. Other than the formerly described TOA and TDOA based LS location estimators, the non-scalable QR decomposition based lateration technique in [4] is also used in the investigation with MOG sensor noise. Furthermore, the performances of TOA and TDOA based location estimators were also compared to the theoretical performance bounds. Theoretical Cramér–Rao lower bounds (CRLB) for TOA and TDOA based location estimation with MOG sensor noise are derived.

The remaining part of the paper is organized as follows. In Section 2, the linearized TOA and TDOA based LS location estimators and the QR decomposition and TOA based location estimator [4] are described. In Section 3, the averaging laterations method of location estimation is described and its performance is investigated by simulation studies. Section 4 considers location estimation when the distance measurements have MOG noise contamination. In this section, first a robust version of averaging laterations is proposed then the performance of this method is compared to simple averaging laterations and single lateration. Section 5 includes a discussion on the proposed averaging technique, draws conclusions from the work and suggests directions of further study. Lastly, in Appendices A–C the derivations of CRLB both for the cases of Gaussian and MOG sensor noise and both for TOA and TDOA measurement data, are given.

2. The localization problem and some linearized estimators

We define a localization setup which will emphasize the main motivation in the paper. So, let us assume that we have N sensors located uniformly on a circle placed at the ceiling and the location to be determined is placed on the floor. Note that this hypothetical placement of the objects does not disturb the applicability of the method in a problem of determining the position of an aircraft using the distance measurements from several base stations like in [11] or determining the location of employees in an office environment like in [22].

The true distance from the ith sensor to the unknown location, $\mathbf{p} = (p_x, p_y, p_z)^T$, can be given as

$$d_{t_i} = \|\mathbf{p} - \mathbf{a}_i\| = \sqrt{(p_x - x_i)^2 + (p_y - y_i)^2 + (p_z - z_i)^2} \tag{1}$$

where $\mathbf{a}_i = (x_i, y_i, z_i)^T$ is the location of the ith sensor and $(\cdot)^T$ denotes the transposition operation. The measurement is modeled as

$$d_i = d_{t_i} + \sigma_i \epsilon_i \tag{2}$$

where ϵ_i is a zero mean Gaussian random variable with unity variance and σ_i is a constant. From a geometrical point of view three and four measurements are required for 2-dimensional (2-D) and 3-D localization, respectively. The distance measurements are generally obtained indirectly, computing the distance traveled by an