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Lossy image compression using singular value decomposition and wavelet difference reduction

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ABSTRACT

This paper presents a new lossy image compression technique which uses singular value decomposition (SVD) and wavelet difference reduction (WDR). These two techniques are combined in order for the SVD compression to boost the performance of the WDR compression. SVD compression offers very high image quality but low compression ratios; on the other hand, WDR compression offers high compression. In the Proposed technique, an input image is first compressed using SVD and then compressed again using WDR. The WDR technique is further used to obtain the required compression ratio of the overall system. The proposed image compression technique was tested on several test images and the result compared with those of WDR and JPEG2000. The quantitative and visual results are showing the superiority of the proposed compression technique over the aforementioned compression techniques. The PSNR at compression ratio of 80:1 for Goldhill is 33.37 dB for the proposed technique which is 5.68 dB and 5.65 dB higher than JPEG2000 and WDR techniques respectively.

1. Introduction

Most applications in human lives such as medicine, ecommerce, astronomy, and remote sensing deal with enormous amounts of digital images [1–5]. This has led to the sharing and storage of large amounts of digital images. The amount of data needed to represent digital images makes transmission slow and storage expensive. The amount of data used to represent these images therefore needs to be reduced.

Image compression deals with reducing the number of bits needed to represent an image by removing redundant data. Psychovisual redundancy takes advantage of the fact that the human eyes ignore some data [6], coding redundancy uses codewords to represent the statistics of the original data while interpixel redundancy explores the fact that some pixels in an image have the same or almost the same value [7,8]. Image compression is broadly classified into two categories, namely lossless and lossy, depending on whether the original image can be recovered with full mathematical precision from the compressed image [7]. In lossless techniques, the original image can be recovered perfectly from the compressed image [8]. In lossy techniques, the original image cannot be recovered from the compressed image as some quantization losses are encountered during the encoding of the image [9–12].

JPEG2000 is a high performance image compression technique developed by the Joint Photographic Experts Group committee. JPEG2000 is based on the discrete wavelet transform and also uses 'tiling' which refers to the partitioning of the original image into rectangular non-overlapping blocks (tiles), which are compressed independently, as though they were entirely distinct images [13,14]. Tiling reduces memory requirements and also improves the compression performance which makes JPEG2000 a state of the art image compression technique. However, other simple algorithms like the wavelet difference reduction (WDR) algorithm can be used as an alternative to JPEG2000 while achieving comparable performance [15].

In this paper a new image compression technique which uses singular value decomposition (SVD) and WDR compression techniques is proposed. SVD is a lossy compression technique which achieves compression by using a smaller rank to approximate the original matrix representing an image [16]. The WDR is also compression technique which is based on the wavelet difference algorithm [17–20]. In our proposed technique, the SVD and WDR methods were combined to make a lossy image compression. SVD compression technique is combined with WDR compression to complement each other. This is because SVD compression technique offers very good PSNR values but very low compression ratios [8,21,23–26]. On the other hand, WDR compression technique offers very good PSNR values at very high compression ratios. However, the performance of the WDR technique can be improved by combining WDR technique with SVD image compression

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Fig. 1. (a) Original Lena image; Lena's image of size 256 × 256 reconstructed by Eq. (4) using (b) 128 singular values, (c) 64 singular values and (d) 32 singular values.

technique. The proposed image compression technique has been tested on well-known images like, Airfield, Boats, Goldhill, Lena, and Peppers and has been compared with the JPEG2000 and WDR techniques.

The quantitative experimental results based on PSNR show that the proposed technique outperforms the above mentioned techniques. This paper is organized as follows: In the second section, we discuss the overview of the methodologies used in our proposed image compression technique in which SVD and WDR are given in details. Then in the third section the proposed lossy image compression is introduced and discussed in details. The experimental results and discussions are given in Section 4 followed by conclusion in the last section.

2. Overview of SVD and WDR

2.1. Singular value decomposition

An image is actually a matrix of numbers whose elements are the intensity value of corresponding pixels of the image. Singular value decomposition is used in order to decompose a given matrix into three matrices known as, U, Σ, and V in which U and V are orthogonal and Σ is a diagonal matrix containing the sorted singular values of the input matrix in descending order [21,22]. Eq. (1) is showing the size of the U, Σ, and V matrices for a given $m \times n$ input matrix.

$$A_{m\times n} = U_{m\times m}\Sigma_{m\times n}(V_{n\times n})^T \tag{1}$$

The number of non-zero elements on the diagonal of Σ determines the rank of the input matrix. Compression is done by using a smaller rank Σ obtained by eliminating small singular values (σ_i) to approximate the original matrix. Mathematically it can be describe as follows:

$$\Sigma_{m\times n} = \begin{bmatrix} \bar{\Sigma}_{p\times q} & 0 \\ 0 & \ddots \end{bmatrix} \quad p \leqslant m \text{ and } q \leqslant n \tag{2}$$

As $\bar{\Sigma}$ has less row and column with respect to Σ thus some column of U and rows of V need to be reduced in order to be able to conduct the matrix multiplications for reconstructing the image, as shown in Eq. (3):

$$U_{m\times m} = [\,\bar{U}_{m\times p} \quad \tilde{U}_{m\times(m-p)}\,] \quad \text{and}$$
$$V_{n\times n} = [\,\bar{V}_{n\times q} \quad \tilde{V}_{n\times(n-q)}\,] \tag{3}$$

Hence the reconstructed matrix can be obtained by

$$A_{m\times n} = \bar{U}_{m\times p}\bar{\Sigma}_{p\times q}(\bar{V}_{n\times q})^T \tag{4}$$

Because the singular matrix has sorted singular values (in descending order) by using the physcovisual concept, ignoring low singular value will not significantly reduce the visual quality of the image. Fig. 1 is showing Lena's picture being reconstructed by using different amount of singular values.

This characteristic that an image can be reconstructed by fewer amounts of singular values takes SVD suitable for compression. Because after reconstruction of the image the ignored singular values cannot be recovered, the compression by SVD is lossy.