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Reduction of quantization noise via periodic code for oversampled input signals and the corresponding optimal code design

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ABSTRACT

This paper proposes to reduce the quantization noise using a periodic code, derives a condition for achieving an improvement on the signal to noise ratio (SNR) performance, and proposes an optimal design for the periodic code. To reduce the quantization noise, oversampled input signals are first multiplied by the periodic code and then quantized via a quantizer. The signals are reconstructed via multiplying the quantized signals by the same periodic code and then passing through an ideal lowpass filter. To derive the condition for achieving an improvement on the SNR performance, first the quantization operator is modeled by a deterministic polynomial function. The coefficients in the polynomial function are defined in such a way that the total energy difference between the quantization function and the polynomial function is minimized subject to a specification on the upper bound of the absolute difference. This problem is actually a semi-infinite programming problem and our recently proposed dual parameterization method is employed for finding the globally optimal solution. Second, the condition for improving the SNR performance is derived via a frequency domain formulation. To optimally design the periodic code such that the SNR performance is maximized, a modified gradient descent method that can avoid the obtained solution to be trapped in a locally optimal point and guarantee its convergence is proposed. Computer numerical simulation results show that the proposed system could achieve a significant improvement compared to existing systems such as the conventional system without multiplying to the periodic code, the system with an additive dithering and a first order sigma delta modulator.

1. Introduction

Quantization is widely employed in many signal processing systems, such as in data compression [1] and analog to digital conversion [2] systems. However, as the quantization is not a reversible process because it is a many to one mapping, signals cannot be perfectly reconstructed after the quantization [3]. As a result, efficient methods for the reduction of the quantization noise are very useful for many signal processing applications.

The most common method to minimize the quantization noise is to perform the quantization based on the statistics of input signals [4]. Finer resolutions are assigned to the ranges of input signals which occur most frequently, and vice versa. However, this kind of quantization schemes requires a prior knowledge of the statistics of input signals. In many situations, the statistics of input signals are unknown and this method cannot be applied directly.

Dithering is also a common method for reducing the quantization noises. However, it worth noting that there are two main fundamental differences between the system with an additive dithering and the proposed system. First, in the system with an additive dithering, a white noise is added and subtracted before and after the quantizer, respectively. On the other hand, a periodic code is multiplied before and after the quantizer in the proposed system. Just changing addition to multiplication will require a very different analytical technique and come up to a very different result. Statistical analysis of an additive noise can be performed easily and plenty of the existing results can be applied. However, the existing techniques for analyzing multiplicative noise are limited and this problem is theoretically challenging. Second, the introduced noise in the additive dithering approach is a random process, while the periodic code in the proposed system is a deterministic signal.

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Since these two signals are very different in nature, the analysis of these two systems is also very different. In terms of the computational effort and the cost of an implementation, since the implementation of the system with an additive dithering requires a random sequence, it is in general more difficult and costly. This is because it is very difficult and costly to generate a truly random signal with a uniform distribution. On the other hand, since a simple potential divider can be employed for performing the multiplication of signals, the implementation of the proposed system is easier and cheaper compared to the existing ones because the periodic code can be stored in memory and only multiplications are required.

Another approach similar to the dithering is via multiplying the signals before and after the quantization by a pseudorandom binary sequence. However, this method is also different from our proposed method. First, the pseudorandom signal is not optimally designed. On the other hand, the proposed periodic code is optimally designed. Second, the pseudorandom signal is only represented by one bit, while our proposed periodic code is represented by more than one bit. Hence, performances of our proposed system are better than those with multiplying the signals before and after the quantization by a pseudorandom binary sequence.

Sigma delta modulation is also widely used to minimize the quantization noise [2,3,5–7,10,11]. If input signals are oversampled, then the signals are bandlimited within a very narrow band [8]. By a proper design of the loop filter, the quantization noise can be further shaped away from the signal band. Although this method can sometimes achieve very high SNRs, many high order sigma delta modulators suffer from the instability problem particularly when the input magnitudes are close to the saturation level of the quantizer [12–14].

In order to reduce the quantization noise with the guarantee of the stability without the prior knowledge on the statistics of input signals, this paper proposes to multiply signals before and after the quantization by a periodic code. Here, the periodic code means a periodic sequence. Periodic codes are widely employed in spread spectrum communication systems. The motivation of the use of the periodic code is based on the fact that the conventional system without multiplying by the periodic code is actually a particular case of the proposed system when the periodic code is equal to one and the period of the code is also equal to one. Hence, the proposed system is the generalization of the conventional system and should achieve an improvement on the signal to noise (SNR) ratio performance if the periodic code is designed properly. The working principles of the proposed method are based on the following arguments. A periodic code can be represented using the Fourier series. Multiplying the input signals by the periodic code is equivalent to the weighted sums of the input signals modulated at different harmonic frequencies. If the quantization operator can be modeled by a polynomial function, then the quantizer performs the weighted sums of the multiplications of the coded signals in the time domain. In the frequency domain, the quantizer performs the weighted sums of the convolutions of the coded signals. It is worth noting that the convolutions of the modulated components will result to the signal components with wider bandwidths and shifting their center frequencies to other harmonic frequencies. After multiplying the quantized signals by the same periodic code and passing through a lowpass filter, all signal components centered at the higher harmonic frequencies will be discarded and only the base band signal component is retained. Although aliasing still occurs in the base band, the effect of the aliasing due to these higher order terms in the polynomial can be minimized by a proper design of the periodic code.

In this paper, the input signal is assumed to be oversampled and it is in the discrete time form. Instead of investigating the analog to digital and digital to analog conversions, this paper is

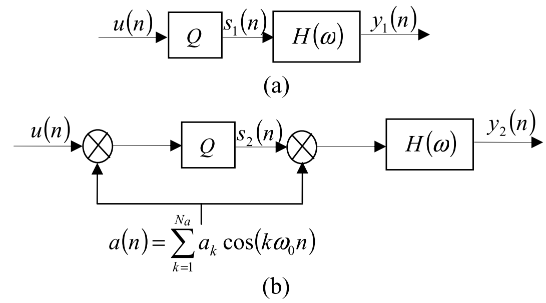

Fig. 1. (a) Conventional system. (b) Proposed system with a periodic code.

to reduce the quantization noise in such a way that the stability of the system is guaranteed without the prior knowledge on the statistics of input signals. To achieve this goal, this paper proposes to multiply the signals before and after the quantization by a periodic code. The outline of this paper is as follow. In Section 2, an approximated model for the quantizer is introduced. Based on the approximated model, detail noise analysis including the derivation of a condition for achieving an improvement on the SNR performance is presented in Section 3. In Section 4, a modified gradient descent method is proposed for designing a periodic code such that the SNR performance is maximized. The proposed method can avoid the obtained solution to be trapped in a locally optimal point and guarantee the convergence of the proposed algorithm. In Section 5, numerical computer simulation results are presented. Finally, a conclusion is drawn in Section 6.

2. Approximated quantization model

It is assumed in many quantization systems that the quantization noise is modeled by an additive wide sense stationary white noise source. The input of the quantizer is also assumed to be a stationary random process. Each sample of the quantization error is assumed to be uniformly distributed over the range of the quantization step size and uncorrelated to the input of the quantizer. Recently, the histogram of the quantizer output is derived analytically based on nonlinear system theories [10]. This result verifies that the assumptions made in the conventional system are invalid and far from practical situations especially for low bit quantizer cases [10]. Hence, a deterministic model, instead of a statistical model, is proposed in this paper.

The block diagrams of a conventional system and the proposed system are shown in, respectively, Fig. 1a and Fig. 1b. Denote the input of these two systems, the quantizer, the frequency response of the linear time invariant filter, the output of the conventional quantizer, the output of the quantizer of the proposed system, the output of the conventional system and the output of the proposed system as, respectively, $u(n)$, $Q(\cdot)$, $H(\omega)$, $s_1(n)$, $s_2(n)$, $y_1(n)$ and $y_2(n)$. We assume that $u(n)$ is oversampled. That means, $u(n)$ is bandlimited within the frequency spectrum $(-\frac{\pi}{R}, \frac{\pi}{R})$, where R is the oversampling ratio (OSR). We also assume that $H(\omega)$ is an ideal lowpass filter. That is,

$$H(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{R}, \\ 0 & \text{otherwise}. \end{cases}$$

Consider an N bit uniform antisymmetric quantizer with the quantization range $[-L, L]$. That is,

$$Q\left(\mu(n)\right) \equiv \begin{cases} \Delta\, sign(\mu(n))(ceil(\frac{|\mu(n)|}{\Delta}) - \frac{1}{2}) & |\mu(n)| \leqslant L, \\ sign(\mu(n))L & |\mu(n)| > L, \end{cases} \tag{1}$$

where $\mu(n)$ is the input of the quantizer,

$$sign\left(\mu(n)\right) \equiv \begin{cases} \frac{\mu(n)}{|\mu(n)|} & \mu(n) \neq 0, \\ 0 & \mu(n) = 0, \end{cases}$$