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Regional sensitivity analysis of aleatory and epistemic uncertainties on failure probability

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ABSTRACT

To analyze the effects of specific regions of the aleatory and epistemic uncertain variables on the failure probability, a regional sensitivity analysis (RSA) technique called contribution to failure probability (CFP) plot is developed in this paper. This RSA technique can detect the important aleatory and epistemic uncertain variables, and also measure the contribution of specific regions of these important input variables to failure probability. When computing the proposed CFP, the aleatory and epistemic uncertain variables are modeled by random and interval variables, respectively. Then based on the hybrid probabilistic and interval model (HPIM) and the basic probability assignments in evidence theory, the failure probability of the structure with aleatory and epistemic uncertainties can be obtained through a successive construction of the second-level limit state function and the corresponding reliability analysis. Kriging method is used to establish the surrogate model of the second-level limit state function to improve the computational efficiency. Two practical examples are employed to test the effectiveness of the proposed RSA technique, and the efficiency and accuracy of the established kriging-based solution.

1. Introduction

In engineering design and risk assessment, the sensitivity analysis (SA) aims to ascertain how the model input factors affect the output, and guides engineers to analyze, forecast and optimize the structural system with uncertainty [1,2]. During the past few years, SA, especially global sensitivity analysis (GSA), has been widely developed and used in engineering applications.

GSA focuses on measuring the contribution of the input uncertainty to the model output by exploring the whole distribution range of the model inputs [2]. At the present time, many GSA analysis techniques have been proposed based on each context: Saltelli et al. [3] and Helton [4,5] discussed the non-parametric methods, Sobol [6,7], Saltelli et al. [8] and Rabitzet et al. [9,10] established the theoretical and numerical background for the variance-based importance measure, Borgonovo [11] put forward the definition of moment independent measures [12,13], and Liu and Chen [14] proposed a relative entropy based on GSA method that studied the impact of an input variable on either the whole or partial distribution range of a response.

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All these GSA techniques cannot tell which part in the range of the important input variables contributes most to the model output, i.e., they cannot identify the intra-variable importance. However, it is especially crucial to identify the important region of an input variable in engineering, since it can provide guidance to the engineer how to deal with the input variables to reduce the output uncertainty. To cope with this problem, many researchers have begun to develop the regional sensitivity analysis (RSA). In 1993, Sinclair [15] proposed the contribution to the sample mean (CSM) plot which was further developed by Bolado-Lavin et al. [16]. The RSA can identify the contribution of specific regions of the input variable to the mean value of the model output. In light of this, Tarantola et al. [17] extended the CSM to the RSA of the input variables on the variance of the model output called contribution to sample variance (CSV) plot.

All the GSA techniques discussed above only considered problems with aleatory uncertainty. However, the available data are frequently limited and of poor quality in engineering applications [18–20]. So the uncertain structure usually contains not only the aleatory uncertain variables but also the epistemic uncertain variables. Sankararaman and Mahadevan [21] recently proposed a GSA technique to separate the contributions of variability and distribution parameter uncertainty to the overall uncertainty. In order to analyze the effects of specific regions of the two types of uncertainty on the failure probability which is paid more attention to in reliability analysis and reliability-based design, a new RSA technique called contribution to the failure probability (CFP) plot, which can be seen as the extension of CSM [16] and CSV [17], is proposed in this paper.

The proposed RSA can provide useful information for reliability design and optimization directly. In this paper, aleatory uncertainty is modeled as random variables by probability theory, and epistemic uncertainty is modeled as interval variables by evidence theory. Combining the hybrid probabilistic and interval model (HPIM) [22] with unified uncertainty analysis [23,24], a HPIM-based analysis approach for the aleatory and epistemic uncertainties is established to compute failure probability of the uncertain structure. In order to improve the computational efficiency, a kriging-based solution is proposed to solve CFP in this paper. This solution employs the kriging surrogate method to fit the second-level limit state function, which is established by analyzing the non-probabilistic reliability index of the structure with random and interval variables.

The remainder of this paper is organized as follows: In Section 2 the unified uncertainty analysis model and HPIM are reviewed, and then the HPIM-based unified uncertainty analysis approach is established to analyze the two types of uncertainty. In Section 3 the CSM and CSV plots are briefly reviewed. In Section 4 the RSA with aleatory and epistemic uncertainties on the failure probability is established, and the direct computational solution and kriging-based solution are proposed for calculating this sensitivity. Two engineering examples are tested in Section 5 to demonstrate the effectiveness, accuracy and efficiency of the proposed kriging-based solution. Section 6 gives conclusions.

2. Unified uncertainty analysis based on the HPIM

Let a limit state function of the input variables in the presence of aleatory and epistemic uncertainties be expressed as

$$G(\mathbf{Z}) = g(\mathbf{X}, \mathbf{Y}) \tag{1}$$

where $\mathbf{X} = \{X_1, X_2, \cdots, X_{n_X}\}$ are the aleatory uncertain variables and treated as independent random variables according to probability theory; $\mathbf{Y} = \{Y_1, Y_2, \cdots, Y_{n_Y}\}$ are the epistemic uncertain variables and modeled by evidence theory with basic probability assignments (BPA). Fig. 1 is an illustration of the epistemic uncertainty with two variables Y_1 and Y_2.

It can be seen from Fig. 1 that the epistemic variables Y_1 and Y_2 contain three and four mutually exclusive focal elements, respectively. The joint BPA of the two epistemic variables is listed in Table 1, where $C_{\mathbf{Y}_i}(i = 1, 2, \cdots, 12)$ is the ith focal element of $C_{\mathbf{Y}}$. In this example, the total number of the focal elements is 12.

According to probability theory and evidence theory [25–27], the unified reliability analysis with aleatory and epistemic uncertainties was established by Du [23,24] and the failure probability P_f can be obtained as

$$\begin{aligned} P_f &= \sum_{i=1}^{n} Pr\{G(\mathbf{Z}) < 0 | \mathbf{Z} \in C_{\mathbf{XY}_i}\} Pr\{\mathbf{Z} \in C_{\mathbf{XY}_i}\} \\ &= \sum_{i=1}^{n} Pr\{G(\mathbf{X}, \mathbf{Y}_i) < 0 | \mathbf{Y}_i \in C_{\mathbf{Y}_i}\} Pr\{\mathbf{Y}_i \in C_{\mathbf{Y}_i}\} \\ &= \sum_{i=1}^{n} m_{\mathbf{Y}}(C_{\mathbf{Y}_i}) P_{f\mathbf{Y}_i} \end{aligned} \tag{2}$$

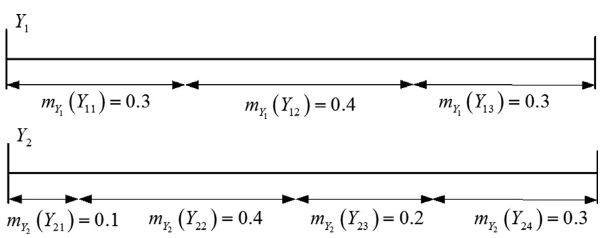

Fig. 1. The BPA structure of the epistemic variables Y_1 and Y_2.