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Nonlinear structural modification and nonlinear coupling

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ABSTRACT

Structural modification methods were proved to be very useful for large structures, especially when modification is local. Although there may be inherent nonlinearities in a structural system in various forms such as clearances, friction and cubic stiffness, most of the structural modification methods are for linear systems. The method proposed in this work is a structural modification/coupling method developed previously, and extended to systems with nonlinear modification and coupling. The method is most useful for large linear structures with nonlinear local modification or coupling. It is based on expressing nonlinear internal force vector in a nonlinear system as a response level dependent "equivalent stiffness matrix" (the so-called "nonlinearity matrix") multiplied by the displacement vector, through quasilinearizing the nonlinearities using describing functions. Once nonlinear internal force vector is expressed as a matrix multiplication form then several structural modification and/or coupling methods can easily be used for nonlinear systems, provided that an iterative solution procedure is employed and convergence is obtained. In the proposed approach the nonlinear FRFs of a modified/coupled system are calculated from those of the original system and dynamic stiffness matrix of the nonlinear modifying system. Formulations and sample applications of the proposed approach for each of the following cases are given, nonlinear modification of a linear system with and without adding new degrees of freedom, and elastic coupling of a nonlinear subsystem to a main linear system with linear or nonlinear elements. Case studies are given for the verification of the method and then a real life application of the method is presented.

1. Introduction

Over the last five decades or so, the finite element (FE) method has established itself as the major tool for the dynamic response analysis of engineering structures. However, for the dynamic reanalysis of large engineering structures modified locally, constituting a FE model each time is expensive and time consuming, especially when several alternatives are to be studied. Therefore, it will be more practical to predict the dynamic behavior of a modified structure by using dynamic response information of an original structure and the dynamic properties or dynamic response of a modifying structure. Various linear structural modification methods have been developed in order to reduce the effort involved in the dynamic reanalysis of modified linear systems. Generally two different problems are considered [1,2], the direct structural dynamic modifications problem and the inverse structural dynamic modifications problem. The inverse structural dynamic modification problem focuses on the determination of the necessary modifications in order to achieve required dynamic characteristics. Some of the main literature reviews on this subject are the studies performed by Li and He [3], Park et al. [4,5], Mottershead et al. [6] and Kyprianou et al. [7,8]. On the other hand, direct structural dynamic modification problem, basic theory of which was primarily presented by Crowley et al. [9], deals with the estimation of dynamic characteristics of a structure after a modification takes place. Some of the most prominent studies on this subject are those from Özgüven [10], D'Ambrogio and Sestieri [11–13] and

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Hang et al. [14–16]. The approach presented in this paper is based on the FRF based matrix inversion technique developed by Özgüven [10] for direct structural modifications.

Although structural modification methods based on linearity assumption are available in literature, these methods cannot be used directly when there is nonlinearity in the system. Nonlinearity is a frequently encountered property of engineering structures which sometimes cannot be ignored in the design of engineering systems. Analysis of nonlinear systems requires a way of handling the nonlinear terms introduced into the equations of motion by nonlinear elements, which are inherently found in structural assemblies formed by many components [17]. Many techniques exist for determining the response of nonlinear systems that are subjected to periodic excitation. Some of these techniques use the Nonlinear Normal Modes (NNMs) concept when dealing with nonlinearities [19–21] whereas some others use the Describing Function Method (DFM) [22–25]. The concept of NNMs was first introduced by Rosenberg [18] and subsequently further developed in recent years by Vakakis, Kerschen and others [19–21]. In this paper, the DFM is used to handle nonlinearities. The DFM is frequently used to quasilinearize nonlinear terms in equations of motion of a nonlinear system [22–25]. It was introduced by Krylov and Bogolyubov [26] in order to analyze certain nonlinear control problems based on an earlier work of Van der Pol [27]. The main idea underlying the DFM is modeling and studying nonlinear system behavior by treating each nonlinear element as a quasilinear descriptor or describing function whose gain is a function of input amplitude [28].

During the past two decades, several structural modification/coupling methods have been suggested taking the nonlinear effect into account. Watanabe and Sato [29] developed the so-called “Nonlinear Building Block” approach for coupling linear substructures with nonlinear joints. They substituted nonlinearities by their equivalent first order describing functions [30] and used FRFs of linear substructures to obtain nonlinear FRFs of coupled structure. Ferreira and Ewins [31] proposed a new method, called the Harmonic Nonlinear Receptance Coupling method, for fundamental harmonic analysis based on describing functions. This method is capable of coupling linear structures with local nonlinear elements whose describing functions are available considering just the fundamental frequency. Then, Ferreira [32] extended this approach and introduced the Multi-Harmonic Nonlinear Receptance Coupling method. This method is able to couple linear and nonlinear structures with different types of linear and nonlinear joints via substituting internal nonlinear forces by their corresponding multi-harmonic describing functions. Chong and İmregün [33] suggested an iterative algorithm for coupling nonlinear systems with linear ones by constructing modal parameter variations of the coupled system from those of its subsystems. Huang [34] also proposed a mathematical model that is capable of predicting dynamic responses of a complex structural assembly considering nonlinearity at the joints via FRF coupling combined with the Harmonic Balance Method.

In this paper, an approach for dynamic reanalysis of large linear structures, either modified locally with a nonlinear substructure or coupled with a nonlinear substructure by using linear and nonlinear coupling elements, is presented. The method suggested [35] is an extension of the method developed by Özgüven [10] for structural modifications of linear systems and differs from the existing methods in that the nonlinear FRFs of a modified/coupled system are calculated from those of the original linear system and dynamic stiffness matrix of the local nonlinear modification/coupled structure. As the dynamic stiffness matrix of a nonlinear system is expressed in this approach in terms of response amplitude, an iterative solution is used. Yet, the method is very powerful for large systems with local modifications. Furthermore, the proposed method is FRF based, and only the FRFs for the DOFs of interest in the original system, in addition to those of the connection points, are used in computations, which reduces the computational time further.

2. Theory

2.1. Structural modification in linear systems

The structural modification method proposed by Özgüven [10] more than two decades ago can be used for structural modification and coupling of linear systems. The method is for dynamic reanalysis of systems where there is a modification in the mass, and stiffness and/or damping of the system. When modifications does not introduce new degrees of freedom (DOFs) to a system, the FRFs of the modified system have been calculated from those of the original system and the dynamic stiffness matrix representing the modifications made by using the following equations:

$$\mathbf{H}^*_{aa} = (\mathbf{I} + \mathbf{H}_{aa}\mathbf{Z}_{\text{mod}})^{-1}\mathbf{H}_{aa} \tag{1}$$

$$\mathbf{H}^{*T}_{ab} = \mathbf{H}^*_{ba} = \mathbf{H}_{ba}(\mathbf{I} - \mathbf{Z}_{\text{mod}}\mathbf{H}^*_{aa}) \tag{2}$$

$$\mathbf{H}^*_{bb} = \mathbf{H}_{bb} - \mathbf{H}_{ba}\mathbf{Z}_{\text{mod}}\mathbf{H}^*_{ab} \tag{3}$$

$$\mathbf{H}^* = \begin{bmatrix} \mathbf{H}^*_{aa} & \mathbf{H}^*_{ab} \\ \mathbf{H}^*_{ba} & \mathbf{H}^*_{bb} \end{bmatrix} \tag{4}$$

here subscript a denotes the coordinates on which a modification is applied and subscript b represents the remaining coordinates of the original system. $\mathbf{H}$ and $\mathbf{H}^*$ are the receptance matrices of the original and modified systems, respectively. $\mathbf{Z}_{\text{mod}}$ denotes the dynamic stiffness matrix of the linear local modification which can be written as

$$\mathbf{Z}_{\text{mod}} = \mathbf{K}_{\text{mod}} - \omega^2\mathbf{M}_{\text{mod}} + j\omega\mathbf{C}_{\text{mod}} + j\mathbf{D}_{\text{mod}} \tag{5}$$