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Identifying parameters of multi-degree-of-freedom nonlinear structural dynamic systems using linear time periodic approximations

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ABSTRACT

The authors recently presented a new nonlinear system identification method, here dubbed the NL-LTP method, in which the system of interest is forced harmonically so that it responds in a stable periodic orbit, and then it is perturbed slightly and its response is recorded as it returns to the orbit. Under mild assumptions the response about the periodic orbit can be approximated using a linear time periodic system model, which can be identified from the measurements using techniques that are akin to linear modal analysis. While the prior work focused on simulated measurements from single degree-of-freedom systems, this work presents several tools that are needed in order to use this approach on multi-degree-of-freedom systems and focuses on applying the method to experimental hardware. The proposed system identification methodology is unique in that it identifies both the order of the nonlinear system and a mathematical model for the nonlinear restoring forces without assuming the mathematical form for the nonlinearities a priori. Towards these ends, this work explains how to extract the underlying nonlinear system model, or nonlinear restoring force versus displacement relationships, from the time periodic model that governs deviations of the system from its periodic orbit, and presents various metrics that can be used to determine which terms in the model are meaningful. These new tools are used to apply the identification method to a continuous, multi-degree-of-freedom structure with a discrete geometric nonlinearity, using both simulated and experimental measurements. The experimental hardware consists of a cantilever beam with a nonlinear spring attached to its tip, which is driven in a periodic limit cycle by an electromagnetic shaker.

1. Introduction

Most dynamical systems behave nonlinearly in the most general scenario. This can be observed in various structural dynamic systems such as an airplane wing that flutters wildly near a stall point bifurcation [1], an automobile shock absorber with nonlinear damping properties [2], in rotor dynamic systems with bearing contact nonlinearities [3], or in biomechanical systems such as the human body which has nonlinear muscles that move joints through large angles [4–6].

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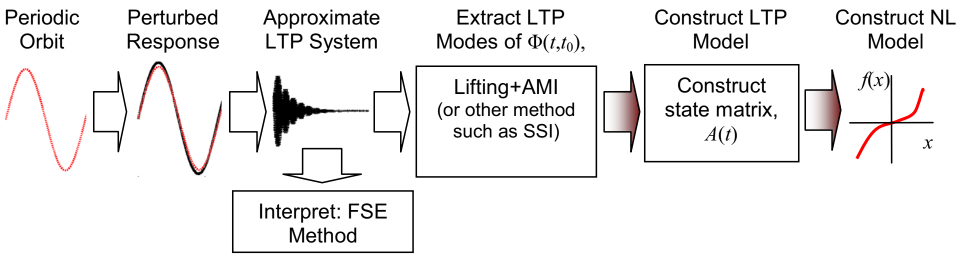

Fig. 1. Overview of the NL-LTP nonlinear system identification technique [14].

While many techniques are available to extract linear time invariant mathematical models from experimental measurements of systems such as these, nonlinear models are needed to correctly characterize some of the complex phenomena exhibited by these systems. Often the form of the nonlinear dynamic model is unknown or poorly known, so an ideal method would identify both the mathematical form of the model and the system's parameters from measurements.

Several methods for nonlinear system identification are currently available. The restoring force surface method [7] and the Hilbert and Hilbert–Huang transforms [8], which are both time domain methods, have been widely used for single degree-of-freedom (DOF) systems. Other methods are more readily applied to multi-degree-of-freedom (MDOF) systems such as, in the time domain, the Nonlinear Auto-Regressive Moving Average with the eXogeneous input (NARMAX) method and the Nonlinear Subspace Identification method [9], and in the frequency domain, the Conditioned Reverse Path and the Nonlinear Identification through Feedback of the Output methods. Many of these methods have the advantage that they not only estimate the nonlinear parameters but they also identify the underlying linear natural frequencies of the system. However, it is important to note that these methods will generally not succeed unless one can assume a correct form for the nonlinearity in advance, and one usually does not have this information for general systems of interest. A few recent review papers present more information on these as well as several other nonlinear identification methods [10–13].

The authors recently proposed an alternative that overcomes some of the limitations of the existing approaches [14]. It is a frequency domain technique based on spectra that are similar to linear frequency response functions. Because of this similarity, the spectra can be curve fit or visually interrogated using the same approaches that are used for linear systems. The order of the system can be determined by counting the spectral peaks or, for example, using more advanced methods based on the rank of the spatial information contained in the spectra. Another advantage of this approach, and one that makes it distinct from many other current methods, is that it does not require an a priori assumption regarding the form of the nonlinearities. It only presumes that a stable periodic orbit exists and that the nonlinearities are differentiable everywhere along the periodic orbit. An overview of the method is provided by the flow chart in Fig. 1, which is adapted from Ref. [14]. The basic idea is to drive the nonlinear system so that it responds in a stable periodic orbit and then to perturb the system slightly from the periodic orbit. If the perturbed response remains close to the original periodic orbit and if the nonlinearities are differentiable along the periodic orbit, then the equations of motion that govern the deviations from the orbit can be well approximated as linear time periodic (LTP). An LTP model can then be identified from the measurements using techniques that capitalize on the linearity of the resulting system (about the periodic orbit), for example the *lifting* and *Fourier series expansion* (FSE) methods, which were both described by Allen in Ref. [15]. The linear time periodic model is used to construct the state transition matrix and state coefficient matrix for each state along the original periodic orbit, which can then be used to estimate the nonlinear parameters of the model. The method was verified for single-degree-of-freedom (SDOF) systems in Ref. [14], where the effect of the periodic orbit on the identification was explored in detail. Here this method shall be referred to as the NL-LTP identification method.

This paper presents several extensions that aid in applying the NL-LTP method to MDOF systems. The most problematic aspect of the NL-LTP identification methodology is the second to last step on the flow chart in Fig. 1 in which the state matrix $A(t)$ is extracted from the modes of the linear time periodic system. Hence, this work focuses on this step, presenting metrics that can be used to check the accuracy of the time periodic modes in order to assure that this final step can be solved successfully. This work also addresses the last item in the flow chart: deriving a method for estimating the individual restoring accelerations that act between different degrees of freedom in the system. These accelerations are equivalent to the restoring forces in the system, scaled by the system mass. All of these tools are used to apply the proposed system identification method, for the first time, to experimental measurements from a cantilever beam with a nonlinear spring attached to its free end. The procedure is first simulated using a computational model, to elucidate the effect of noise on the NL-LTP method.

The following section reviews the theory that supports this approach, some of which was presented in Ref. [14]. The enhancements mentioned in the preceding paragraph are developed in Section 2.3. The nonlinear beam that is used in the experimental setup is presented in Section 3 and its analytical model described. The proposed identification techniques are then applied to measurements of the beam and the results are presented and discussed. Conclusions are presented in Section 4.