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Modeling energy transport in a cantilevered Euler–Bernoulli beam actively vibrating in Newtonian fluid

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ABSTRACT

When a mechanical and/or structural component is immersed in a fluid and it vibrates, the reasonable assumption is that part of the energy is transmitted to the adjacent media. For some engineering applications the energy transport between these two domains, *i.e.*, structure and fluid, plays a central role. The work presented in this paper is focused on discussing the energy transport in beam-like structures as they can be used to represent flexible swimmers (fish-like pulsating mechanisms) in their simplest form. In order to expose the role of each of the fluid and beam properties effecting the energy transfer process, a simplified analytical fluid–structure interaction (FSI) model is derived. After analysis of the resulting coupled-systems' damping coefficient, a new energy transport component is added to the initial Euler–Bernoulli beam equation; a term associated with diffusion (fluid viscosity). In addition our modeling results in an added mass term, a characteristic consistent with previous literature. While deriving the model, an important assumption is made: beam mode shapes are not significantly affected by the domains' interaction. This hypothesis is experimentally tested in two different fluid media and confirmed to be reasonable for the first three vibration mode shapes.

1. Introduction

Several engineering applications rely on the energy transfer between domains, *e.g.*, thermal energy converted into motion. An understanding on how the transfer occurs empowers designers with appropriate tools and concepts to maximize, or minimize, this process.

The work developed here focuses on a subclass of the fluid–structure interaction (FSI) problem: solid slender-body structures, particularly beams, actively vibrating in incompressible Newtonian fluid, as they can represent propulsion generation in fish-like structures [1], *etc.* The mathematical foundation for this kind of problem was developed by Rayleigh [2,3]. In practice the problem is addressed by ignoring one domain, disregarding coupling effects that could be beneficial for the application itself. Wu [4] ignored the structural domain, assuming that the plate induced a known displacement to the surrounding fluid. Using potential flow theory, he evaluated the pressure distribution and accessed the lift and drag forces on the plate produced by such oscillations.

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Lindholm [5] provided vast experimental data and analytical developments on the structural side of the problem, disregarding the flow patterns created by the plate oscillation. Lindholm's goal was to predict and understand generation of high frequency noise by the structure, a recurrent phenomenon in ship design in the 1950s. The plate equation was modified considering that the fluid added inertia to the system and the apparent mass was calculated based on the pressure distribution around the plate using strip theory. A ratio between the natural frequency of the plate in air and water was derived for each mode shape and results were compared to experimental data. One important observation was the fact that mode shapes in water were fairly similar from the ones observed in air.

In the late 1960s and early 1970s researchers started using the term fluid–structure interaction (later know by the acronym FSI) to describe analysis that would take in account the reciprocal effect on both domains [6]. Several fluid mechanists, interested in understanding fish locomotion, developed elegant coupled analytical models that provided great insight on the physics of the problem [1,7,8]. The flow around the oscillating structure was described in terms of a known unsteady flow potential, which was used in the calculation of a pressure field and then combined to the structure elasticity and inertia. Unfortunately for simplification and comparison reasons, the structural inertia was dropped out of the analysis, *i.e.*, ignoring the fluid–structure resonance phenomenon. Haddara and Cao [9] addressed this issue by lumping the fluid effects into an added mass term to allow an analytical solution.

Following the drastic improvement in computational power, several numerical methods (discrete) were developed to describe complex fluid behaviors. Fu and Price [10] laid out a complete structural and fluid model (independent of each other), where the flow potential was resolved using a panel method and the structure using the finite element method. The more elaborate FSI analysis allowed a discussion on the coupled system damping and pointed out differences between fluid effects on lower and higher mode shapes of the structure.

Discrete methods introduce a gap between domains requiring coupling strategies to be implemented as part of the solution algorithm. Maity and Bhattacharyya [11] describe the most common approaches to the problem and give several references on each. Several other researchers contributed on improving the computational fluid dynamics (CFD) side of the problem [12–14]. Although these methods improve the representation of such complex coupled systems, they do not provide an explicit interpretation of the underlying physics.

In trying to understand the basic mechanism originated from the coupling of both domains and to reduce the computational power required to describe the phenomenon, a large number of papers were published on the analytical development of the problem [15–18]. Despite the insights on added inertia and thrust/drag obtained by these models, few try to discuss the damping introduced by the surrounding fluid and its impact on energy transfer across the domains. Another important aspect of these models (and most FSI research and literature) is that they usually focus on flow-induced vibration disregarding the bi-directionality of the problem, *i.e.*, vibration-induced flows.

Permanent energy exchange was investigated by Tang et.al. [19], who focused on the energy transferred to the structural domain by the fluid, enabling the concept the authors call “flutter mill.” The analysis was based on several numerical simulations each one with different parameters. The design of such energy harvesting devices and other engineered systems relies on an understanding of how the coupling process takes place.

Interested in explaining the interaction between fluid and structural domain, we developed a simple analytical model that captures the most basic features of the phenomenon. The resulting differential equation that represents the coupled system behavior has two new components (when compared to just the structural model): an increase in inertia and energy transport diffusion to the adjacent media, *i.e.*, fluid.

Assuming that structural mode shapes are not affected by the presence of a surrounding fluid is a key concept of the modeling process. Although such hypothesis is generally considered to be valid by several authors in the literature [20], an experimental test was performed to evaluate the cantilevered beam lowers mode shapes and its modal properties.

The novelty of the proposed approach is based on the effects of the fluid motion into the structural damping, a parameter that has been neglected in past studies and that carries important information on the permanent energy exchange across domains.

2. Fluid domain modeling

The evaluation of pressure fields around a body/structure immersed in a fluid that has a prescribed motion is quite a complex problem. Several researchers and engineers appeal to discrete (numerical) methods to obtain such data. Examples can be found in Maity and Bhattacharyya [11]. A less common approach is to develop analytical solutions, which carries more physical meaning, but requires a large number of simplifications, *e.g.*, Haddara and Cao [9].

In order to understand the coupling mechanism between the beam and surrounding fluid a simple analytical model in both domains is required. Fig. 1 provides an illustration of the simplified model used here. An active cantilevered beam, of length (L), is immersed into a fluid. It is important to notice that the beam motion (y_b) is only activated in the y-direction.

The fluid is Newtonian and the flow is considered to be incompressible and two-dimensional, *i.e.*, there is no fluid-velocity component or gradients in the z-direction. The flow over the beam is assumed to have the same vertical (y) velocity as the beam, which can be found by deriving the beam y-position (y_b) with respect to time. Separation of variables is