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Inverse dynamic substructuring using the direct hybrid assembly in the frequency domain

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ABSTRACT

The paper deals with the identification of the dynamic behaviour of a structural subsystem, starting from the known dynamic behaviour of both the coupled system and the remaining part of the structural system (residual subsystem). This topic is also known as decoupling problem, subsystem subtraction or inverse dynamic substructuring. Whenever it is necessary to combine numerical models (e.g. FEM) and test models (e.g. FRFs), one speaks of experimental dynamic substructuring. Substructure decoupling techniques can be classified as inverse coupling or direct decoupling techniques. In inverse coupling, the equations describing the coupling problem are rearranged to isolate the unknown substructure instead of the coupled structure. On the contrary, direct decoupling consists in adding to the coupled system a fictitious subsystem that is the negative of the residual subsystem. Starting from a reduced version of the 3-field formulation (dynamic equilibrium using FRFs, compatibility and equilibrium of interface forces), a direct hybrid assembly is developed by requiring that both compatibility and equilibrium conditions are satisfied exactly, either at coupling DoFs only, or at additional internal DoFs of the residual subsystem. Equilibrium and compatibility DoFs might not be the same: this generates the so-called non-collocated approach. The technique is applied using experimental data from an assembled system made by a plate and a rigid mass.

1. Introduction

The paper deals with the identification of the dynamic behaviour of a structural subsystem, starting from the known dynamic behaviour of both the coupled system and the remaining part of the structural system (residual subsystem). This topic is also known as decoupling problem, subsystem subtraction or inverse dynamic substructuring. Decoupling is a relevant issue for subsystems that cannot be measured separately, but only when coupled to their neighbouring substructure(s) (e.g. a fixture needed for testing or subsystems that are very delicate or in operational conditions).

Inverse substructuring can be seen as a special case of dynamic substructuring, or as a structural modification problem with negative modification [1]. However, whilst many well-established techniques exist to perform subsystem addition when all substructures are modelled theoretically, in subsystem subtraction this would be a trivial problem. Therefore, subsystem subtraction is useful when the model of at least one subsystem derives from experimental tests. In such cases,

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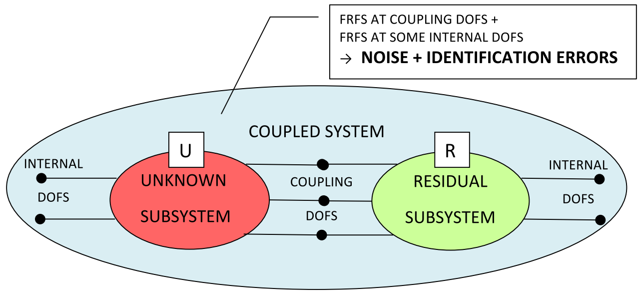

Fig. 1. Scheme of the decoupling problem.

one speaks of experimental dynamic substructuring. Due to modal truncation problems [2], the use of FRFs (Frequency Based Substructuring) is preferred with respect to the use of modal parameters. The main algorithm for frequency based substructuring is the improved impedance coupling [3] that involves just one matrix inversion with respect to the classical impedance coupling technique that requires three inversions. A general framework for dynamic substructuring is provided in [4,5], where the primal assembly and the dual assembly are introduced.

Substructure decoupling techniques can be subdivided into inverse coupling techniques and direct decoupling techniques. In inverse coupling, the equations written for the coupling problem are rearranged to isolate (as unknown) one of the substructures instead of the assembled structure. Examples of inverse coupling are impedance and mobility approaches [6–8].

Direct decoupling consists in adding to the assembled system a fictitious subsystem, which is the negative of the residual subsystem. The technique starts from the 3-field formulation: one set of equations expressing the dynamic equilibrium of the assembled system and, separately, of the fictitious subsystem; one set of equations enforcing compatibility at interface DoFs, one set of equations enforcing equilibrium of constraint forces at interface DoFs. To solve the problem, dual assembly [9], primal assembly [10] or hybrid assembly [11] can be used. Compatibility and equilibrium can be required either at coupling DoFs only (standard interface), or at additional internal DoFs of the residual subsystem (extended interface). Furthermore, DoFs used to enforce equilibrium need not to be the same as DoFs used to enforce compatibility [12]: this gives rise to the so-called non-collocated approach, as opposite to the traditional approach in which such DoFs are the same, which is called collocated. The choice of interface DoFs determines a set of frequencies at which the decoupling problem is ill conditioned, as shown in [9] for the dual assembly. Apparently, when using an extended interface, the problem is singular at all frequencies, although this singularity is easily removed by using standard smart inversion techniques.

In this paper, a reduced version of the 3-field formulation (dynamic equilibrium using FRFs, compatibility and equilibrium of interface forces) is presented in Section 2: this formulation is particularly suited for experimental dynamic substructuring since it uses only measurements on the minimum set of DoFs that are necessary to enforce compatibility and equilibrium conditions. Starting from the 3-field formulation, the dual assembly is revisited and a hybrid assembly is presented by requiring that both compatibility and equilibrium conditions are satisfied exactly, either at coupling DoFs only, or at additional internal DoFs of the residual subsystem. Dual and hybrid assembly are compared both from the theoretical and the practical point of view, using experimental data from an assembled system made by a plate and a rigid mass described in Section 3. Results obtained using collocated and non-collocated approaches are presented and discussed in Section 4.

2. Direct decoupling techniques

The coupled structural system RU (N_{RU} DoFs) is assumed to be made by an unknown subsystem U (N_U DoFs) and a residual subsystem R (N_R DoFs) joined through a number of couplings (see Fig. 1). The residual subsystem (R) can be made by one or more substructures. The degrees of freedom (DoFs) of the coupled system can be partitioned into internal DoFs (not belonging to the couplings) of subsystem U (u), internal DoFs of subsystem R (r), and coupling DoFs (c).

It is required to find the FRF of the unknown substructure U starting from the FRF of the coupled system RU. The subsystem U can be extracted from the coupled system RU by cancelling the dynamic effect of the residual subsystem R. This can be accomplished by adding to the coupled system RU a fictitious subsystem with a dynamic stiffness opposite to that of the residual subsystem R and satisfying compatibility and equilibrium conditions. The dynamic equilibrium of the coupled system RU and of the fictitious subsystem can be expressed in block diagonal format as follows:

$$\begin{bmatrix} \overline{\mathbf{Z}}^{RU} & \mathbf{0} \\ \mathbf{0} & -\overline{\mathbf{Z}}^{R} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{u}}^{RU} \\ \overline{\boldsymbol{u}}^{R} \end{Bmatrix} = \begin{Bmatrix} \overline{\boldsymbol{f}}^{RU} \\ \overline{\boldsymbol{f}}^{R} \end{Bmatrix} + \begin{Bmatrix} \overline{\boldsymbol{g}}^{RU} \\ \overline{\boldsymbol{g}}^{R} \end{Bmatrix} \tag{1}$$

where

- $\overline{\mathbf{Z}}^{RU}$, $\overline{\mathbf{Z}}^{R}$ are the dynamic stiffness matrices of the coupled system RU and of the residual subsystem R, respectively;
- $\overline{\mathbf{u}}^{RU}$, $\overline{\boldsymbol{u}}^{R}$ are the vectors of degrees of freedom of the coupled system RU and of the residual subsystem R, respectively;